CONVENTIONALITY OF SYNCHRONISATION, GAUGE DEPENDENC AND TEST THEORIES OF RELATIVITY

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Physics Reports 295 (1998) 93–180

PHYSICS REPORTS

Conventionality of synchronisation, gauge dependence and test theories of relativity

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Received June 1996; editor: G.H.F. Diercksen

Contents

Abstract

The formal and operational significance of the choice of clock synchronization in relativity is reviewed, along with the historical debate over the associated choice of the one-way speed of light. Kinematic test theories generalising special relativity are recast in a nonstandard synchronisation. In particular, the Mansouri–Sexl test-theory is generalised to avoid a conflict between its interpretation and its gauge choice. Corresponding adjustments to the interpretation of recent experimental tests of relativity are presented. A test-theory for local Lorentz invariance is derived for a noninertial observer in a space of arbitrary curvature using differential geometric techniques and the Frenet frame. The Sagnac effect in a ring laser is considered for bounding the parameters of this theory. © 1998 Elsevier Science B.V.

PACS: 03.30.+p; 04.20.−p; 11.15.−q; 42.60.Da

Keywords: Relativity; Synchronization; Tests; Gauge; Laser

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PII S0370-1573(97)00051-3

1. Introduction

1.1. Background

One of the significant discoveries in the 19th century programme of extending Maxwell's electromagnetic theory was Heaviside's demonstration in 1888 that the electromagnetic field around a moving charge is compressed in the direction of motion by the factor of $\sqrt{1 - v^2/c^2}$ [82]. Heaviside noted his failure to understand what happened physically when $v > c$ and when he wrote to George FitzGerald in February 1889 about his result, FitzGerald noted in reply that he too had puzzled on this point:

> "You ask ... what if the velocity be greater than that of light? I have often asked myself ... is it possible?"

FitzGerald also suggested that the speed of light may be an upper bound for speeds. Further details of this correspondence may be found in [87, 15]. Later in that same year FitzGerald proposed his contraction hypothesis for moving bodies involving the same factor as Heaviside's formula.

Along with this emerging awareness that the speed of light may be an upper bound for communication, arguments were also given that any remote synchronization involved a circular argument. Some were presented in several publications by Poincaré [168–170]. Even earlier, in 1880, Simon Newcomb was apparently aware of the flaws in an experimental proposal to determine the speed of light in one direction. In 1904 Michelson [140] remarked of this experiment and Newcomb's response:

> "In the Physikalische Zeitschrift (5 Jahrgang, No. 19, Seite 585–586) a method is proposed by W. Wien for deciding the important question of the trainment of the aether by the earth in its motion through space, by measuring the velocity of light in one direction ... The essentials in the proposed method are two Foucault mirrors, or two Fizeau wheels (one at each station) revolving at the same speed.... The flaw in the proposed method – as was pointed out by Simon Newcomb as long ago as 1880 – lies in the fact that the effect which it is proposed to measure is exactly the same as the effect on the light which is to furnish the test of synchronism."

In both of these issues as well as others, such as the consideration of the feasibility of first order tests of the speed of light in a given direction (see, e.g., the correspondence between Newcomb and Michelson on this matter as quoted by Reingold [179]), we see the elements that formed the basis in this century of a position that the method by which clocks are synchronized in a spacetime frame of reference has an irreducible element of convention about it. In this study we explore several consequences of this.

The reanalysis of the concept of simultaneity formed one of the crucial and distinguishing elements of Einstein's Special Theory of Relativity of 1905 [48]. Moreover, in 1924, in one of the few places where he commented on the origins of the Special Theory, Einstein remarked that "By means of a revision of the concept of simultaneity in a shapable form I arrived at the special relativity theory" (quoted in [63, p. 176]). Instead of an absolute time associated with the rest frame of the aether, actual time for an observer was that of clocks attached to the inertial frame of the observer. Whether or not events were simultaneous was determined by the readings of clocks at the place of the events.

Such clocks were synchronised by an operational procedure using light signals. The uniqueness of Einstein's understanding of time may be seen by comparing it to that of his contemporaries such as Lorentz (Section 2.1).

The standard case when light propagation is assumed to be isotropic is known as "Einstein synchronization", because this method was proposed by Einstein in the kinematic section of his 1905 paper along the following lines. A signal, from a clock at position A, is sent at a time t_1 (on that clock) to a distant clock at position B and then reflected back to the clock at A, arriving at (local) time t_3. The time of arrival at position B is defined to be the mean of the times t_1 and t_3. Einstein specifically noted [48] (English translation [52, p. 142])

> "*by definition* that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A."

Such "imaginary physical experiments", Einstein remarked, provide a way to understand what is meant by synchronous clocks at different places. Einstein took the round-trip speed, namely $c = 2AB/(t_3 - t_1)$, to be isotropic. (This itself reflects a coordinate or gauge choice, of a type different to that discussed in the bulk of this review; see Section 1.5.4.) Einstein also noted that this synchronization of clocks in a frame will be a symmetric relationship (if clock B is synchronous with clock A, then A will be synchronous with B) as well as a transitive one (if a clock A is synchronous with B as well clock C, then B and C will be synchronous relative to each other) [52, p. 143]. The question of whether a synchronization procedure leads to an equivalence relationship became an issue in later discussions on non-Einsteinian definitions of simultaneity (see Section 2.1.4). Some subtle issues in connection with the extent to which this is understood of a "resting frame" only, and whether it implies the constancy and invariance of the speed of light as well as its isotropy, are analysed by Brown and Maia [24]; see also Sections 1.5.4, 3.1.2. The operational method clearly associates the synchronization within the frame with the velocity of light in the frame. Indeed, Einstein later made this explicit in his 1907 review article [52, p. 256]:

> "We now assume *that the clocks can be adjusted in such a way that the propagation velocity of every light ray in vacuum – measured by means of these clocks – becomes everywhere equal to a universal constant* c, provided that the coordinate system is not accelerated."

In his popular exposition of the special and general theories of relativity [51], Einstein stressed the inherently circular nature of such knowledge: one was prevented from measuring the one-way speed of light in a given direction because that would require the prior synchronization of clocks and thus a prior knowledge of the speed to be measured. The choice that light travels at equal speeds along the opposite directions of a particular path was

> "neither a *supposition nor a hypothesis* about the physical nature of light, but a *stipulation*"

that can be freely made so as to arrive at a definition of simultaneity.

Following this lead, standard developments of the special theory of relativity universally assume a constant and isotropic speed of light in every inertial frame. All derived quantities, such as time dilation and length contraction effects on a moving body as seen from an inertial frame, are also independent of direction.

However, along with standard treatments the possibility in principle of postulating an anisotropic structure has been discussed extensively in various philosophical and physical contexts. Hans

Reichenbach's "The Philosophy of Space & Time" [177] has been most significant in the history of the debate. It brought out and defended the definitionary nature of Einstein's treatment of simultaneity and established a notation for expressing an anisotropy in the one way speed of light. It was first published in German in 1928. In the 1960s and 1970s Adolf Grünbaum [73–75] also defended the need for a definition specifying simultaneity when discussing the one-way speed of light and brought the issue alive within philosophical contexts. Since then the issue has remained a contentious one in both philosophical as well as physical contexts, and of continuing relevance to a variety of foundational issues (see Sections 1.2, 1.3.1, 1.3.2). Physical issues brought new life into the discussion, and new directions and strategies have emerged; in the philosophical context and especially in recent years the debate has suffered by being explored largely in isolation from the details of physical theories. We consider the leading arguments for these strategies and demonstrate the importance of carefully clarifying the conceptual and philosophical issues for a correct understanding of the significance of tests of special relativity.

An extension of these principles with recurring importance is what Reichenbach [178, Section 11] calls "the round trip axiom: If from a point A of the static system two light signals are sent in opposite directions along a closed triangular path ABCA, they will return simultaneously". Reichenbach attributes the recognition of this axiom to Einstein. Weyl [228, Section 23] has a similar axiom, which he refers to as an *Erfahrungstatsache* ('fact of experience'): the round-trip journey around a polygonal path depends only on the length of the path and the round trip speed of light. Weyl's axiom includes Reichenbach's axiom; these axioms are connected by, while distinguishable from, the "light postulate" (that the two-way speed of light is isotropic, Section 1.5.4). (An alternative interpretation of Weyl's *Erfahrungstatsache*, making it more akin to Robertson's approach by denying the light postulate until confirmed by the 'experience' of the Michelson–Morley experiment, is discussed in Section 3.1.2.) Within general relativity, the Reichenbach round-trip axiom forms a condition on the definition of a local inertial frame; it must not only be in free fall but also be non-rotating in this sense, so as to have no Sagnac effect [204]. These extended principles then state that there is no Sagnac effect for any closed path in such a frame. They constrain any non-standard rule for synchronization (Section 2.3.4), and guarantee that any non-standard synchronization defines a unique hypersurface for any set of simultaneous events, so forming an equivalence relationship (Sections 2.1.4, 2.3.4). Moreover, they ensure (as Weyl notes) that the synchronization established throughout a frame from a master clock using light signals will be independent of the location of that clock in the frame – a feature which we note to be true of Einstein synchronization as well as of non-standard synchronizations which form an equivalence relationship.

1.2. The essential issues

How could one disprove the statement: "The speed of light in vacuum from left to right is double that from right to left"? By saying that this violates the isotropy of space? That begs the question. By timing its travel in each direction to check or refute the notion? But in relativity one sets up clocks using light signals assuming that these speeds are the same and are equal to c.

We can measure a round trip time on one master clock M, say at the spatial origin, on whose reading everyone can agree. We take this to measure the round-trip distance d to some remote clock N, as in the modern, light-speed based, definition of the metre. Such round trip considerations show only that the two speeds in the above statement of anisotropy (let us call them $2v$ and v

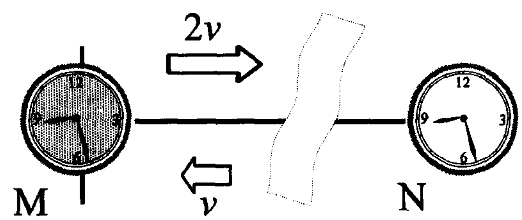


Fig. 1. Remote synchronization of two clocks. Taking the one-way speed of light to be nonreciprocal (say $2v$, v for forth and back respectively) is equivalent to changing the choice of setting at a clock N remote from the master M (shaded).

for the journeys MN and NM, respectively; Fig. 1, must average to c and so must be $\frac{3}{2}c, \frac{3}{4}c$. The original statement has not been refuted, but made more precise; it amounts simply to a setting up of remote timekeeping consistent with this assumption. Any manner of coordinatising time throughout a frame is acceptable, provided it is coherent and internally consistent, and provided the manner in which it represents physical processes (in particular, their causal relationships) is clearly understood.

If one takes the point of view that a metrical labelling of events by coordinates is the most important aspect of time for the physical description of events, any coherent and internally consistent method of keeping time is acceptable.

Indeed, even with instantaneous signalling and so observable verification of remote synchronization, such as that permitted in classical Galilean theory, full coordinate freedom can still be claimed for the theory. This claim has been regarded as sterile, the general covariance of a theory itself having no physical significance [103]. Some view this as unduly extreme (see Sections 1.4.1, 2.5.1). Brown [23] comments that this view is not consistent with the historical importance of the arguments from covariance by Maxwell and Einstein in developing theoretical physics. Ellis et al. [56] and Ohanian et al. [160] emphasise that coordinates are needed for measurement, and that some problems may need definite coordinates. We allude to this coordinate freedom as gauge freedom.

Special relativity adds a unique and characteristic weight to the emphasis of this coordinate freedom, reflecting a decidedly nonsterile aspect of the physics of spacetime. There is inevitably a finite difference $2d/c$ between the times t_E, t_R of emission and reception at the master clock M of a light ray reflected off the remote clock N. Hence the coordinate freedom which already exists is arguably naturally exercised so as to give N any intermediate time in the interval (t_E, t_R). With this restriction the time coordinate labeling will reflect the causal ordering of events connectible by light rays and negative velocities for one-way speeds of light will be avoided. Following an historical custom, we allude to this particular exercise of gauge freedom as the "conventionality of synchronization".

Because we have empirical access only to the round-trip average speed of light, statements about the magnitude and isotropy of the one-way speed of light must reflect the assumptions made in the choice of time coordinatization, and such entities change as the theory is re-coordinatized, or gauge transformed. Other methods of establishing synchrony in a frame do not change the conventional status of these one-way quantities.

We shall see that all speed-dependent expressions, including the parameters of the Lorentz transformation and so (for example) time dilation factors, have irreducibly conventional elements. This conventionality hinges on the existence of an infinity of possible synchronization schemes for the setting of the clocks to be used in the measurement. One cannot single out from these some particular choice without an a priori assumption about the one-way speed of light.

No experiment, then, is a "one-way" experiment. An empirical test of any property of the one-way speed of light is not possible. Such quantities as the one-way speed of light are irreducibly conventional in nature, and recognizing this aspect is to recognize a profound feature of nature.

Any such extension and correction of standard test theories itself cannot avoid another conventional coordinate choice, even if one convention is explicitly exhibited. Those conventional elements relevant to the conclusions that are drawn need to be made explicit. In particular, it is a matter of principle that one cannot "test for the isotropy of the speed of light", for to achieve this would be to contradict the synchrony dependence of a one-way speed. Rather, the insights afforded by the existence of this freedom in coordinate choice are valuable in all contexts as that of the twin paradox where questions of simultaneity are raised [176].

An illustration from electrostatics may help; it is more closely related to our topic than may at first appear (Section 2.3.3). Voltmeters are useful, despite the dependence of the value of the voltage at any point on an arbitrary electromagnetic gauge choice. The conventional content of the voltage concept is well understood and is evidenced by the manufacturers' provision of a second (earth) probe on each voltmeter. We would not counsel the removal of "Danger High Voltage" signs, even though the implied convention allows the re-classification of an electrical feeder wire as being at low voltage. On the other hand, if a company were to advertise a product with just one external probe which purported to "test for the absolute zero of voltage in seawater", the matter becomes more serious than one of taste and judgement. This kind of claim is corrected in the following. Our fundamental aim is to clarify, if by analysis of such counterexamples, those testable facts which are independent of convention (see for example the discussion in Section 1.3.2).

1.3. Recent objections

1.3.1. Philosophy

In the context of philosophy, the discussion centres on the grounds upon which the natural choice of isotropy may be regarded as obligatory. It has often been maintained within the community of philosophers of science that theoretical considerations based on the context and symmetries of the causal structure of Minkowski space–time (by which we mean the conformal structure, the lack of any distinguished orientation, and the relationship between events induced by the propagation of light rays) are sufficient to force the choice of Einstein synchronization upon any reasonable theoretical formulation. While this has been a recurring theme of many articles over many years, it has become more strident subsequent to the work of Malament [131].

Indeed, Friedman [62, p. 310f] claimed that Malament had shown that Einstein synchronization is explicitly and uniquely definable from the conformal structure of the space–time metric, and concluded that dispensing with Einstein synchronization entailed a denial of the structure of Minkowskian space–time. This is not an isolated view. Norton, for example, has viewed the result on the debate on the conventionality of simultaneity as "one of the most dramatic reversals in debates in the philosophy of space and time" [159]. However, such authors as Havas [81], Redhead [175] and Debs et al. [176] have contended with this extreme reading of the significance of Malament's work. This is discussed more extensively later (see Section 2.2.1), with the conclusion that Malament's arguments have been given undue credence in much previous work.

Another approach has been to place conditions on the type of spatial metric one should have on a hyperplane of simultaneity and argue that they can only be satisfied by hyperplanes determined by Einstein synchronization (see, e.g., [37,38] and countering arguments [5,6]).

Questions on the definition of simultaneity continue to be raised in connection with measurement (e.g., the choice of hyperplane of simultaneity for the Copenhagen collapse), nonlocality and quantum indeterminateness and how these are to be understood in the context of the space–time of special relativity theory. For a recent introduction to this discussion, see [43].

1.3.2. Physics

Recently, also in the physics community there have been renewed claims that certain experiments constitute experimental evidence for the isotropy of the one-way speed of light.

This reflects part of a resurgence of interest in experimental tests of general relativity in the 1970s and 1980s, now past its peak but still significant, and now targeted more towards tests of local Lorentz invariance. Many of these tests have been documented by Will [229,231] who has also issued updates [232]. Experiments claimed to give experimental evidence for the isotropy of the one-way speed of light include an experiment at the Jet Propulsion Laboratory, Pasadena, CA, USA, involving the monitoring of the relative phase stability of two remotely linked masers using the Goldstone Deep Space Communications complex (Section 3.3.2), and an experiment at Boulder, CO in two-photon absorption (Section 3.3.1) by a group world-renowned for expertise in stabilising lasers and high-precision spectroscopy. Nelson et al. [151] use many resources of NASA and the US Naval Observatory to claim "an investigation of whether the one-way speeds of light in the east–west and west–east directions on the rotating earth are the same." Krisher et al. [111] suggest the use of data from the Galileo probe "to test possible anisotropies in the velocity of light".

Jammer [89] comments on the fact that such questions on synchrony issues are raised, not by charlatans, but by competent and serious physicists. When one also considers the staggeringly expensive resources that have been used for support of the various experimental claims, we consider the present discussion to be justified if only because the least one may expect is a thoroughly developed theoretical consensus of the interpretation of such experiments. Admittedly, these applications can be ancillary uses of such equipment, as with the tests based on lunar laser ranging [146].

This emphasis on the supposed measurability of one-way properties reflects an approach which has its origins in Robertson's pioneering work on testing special relativity [183]. Robertson's work in turn motivated the Mansouri–Sexl test-theory [133] which for 20 years has provided the most popular framework for analysing relevant experiments. Although Mansouri has given full weight to some aspects of the conventionality of the one-way speed of light [136], this was incompletely incorporated in the test theory. The essential problem is that the effects of convention were analysed in the laboratory frame, but not in the reference or "aether" frame. (This may have its philosophical roots in a particular approach to relativity, which may be traced to Einstein [24], in which he assumes a "principle of light speed constancy" in a "resting" frame, and deduces comparable results in a frame moving with respect to this frame.) As a result, the Mansouri–Sexl analysis of test theories itself embodies an appreciable and indefensible anticonventionalist element, in particular the feasibility of first-order tests of relativity and of gaining empirical evidence on the *actual* one-way speed of light. The correct interpretation of the Mansouri–Sexl test theory was thus obscured by its authors at the very start of its historic and continuing reign (see Section 3.1.4). A later emphasis on experiments for the measurement of the "one-way speed", starting with the empirical approach of Vessot [220] (we note some correspondence in *Physics Today* [105, 106]), reflects these assertions by Mansouri and Sexl; it gained prominence through the work of Krisher et al. [110], Will [229, 230] and Haugan et al. [78], who titled their papers on the lines "Testing the isotropy of the one-way speed of light ...".

A side comment is necessary on this nomenclature. The concept of *isotropy* of the speed of light – strictly, its insensitivity to any change in direction – is not initially equivalent to that of the *reciprocity* of that speed – its equality for forward and reverse traverse of any path. These become equivalent statements when the apparent round-trip speed of light is accepted to be c (as do virtually all authors, and as we do in the body of this article). There is then no need to differentiate between the capabilities of different experiments on the basis of such distinctions. Two attempts to enforce such a distinction in this context, which in our judgement are abortive, are discussed in Sections 3.1.2 and 3.3. However, one can certainly find coordinate transformations which make the round-trip speed of light other than c (see Section 1.5.4), and in such theories one would have to make such distinctions. We also note that only vacuum speeds are under consideration in this article; for synchrony to be defined by material and possibly nonreciprocal speeds of light is a complication which is of no interest in our context.

All such experiments in fact focus on comparisons of the *apparent* speeds of light in opposite directions on a given spatial path *as determined from the synchronization defined by a slowly transported clock*. The proper interpretation of these experiments is that slow clock transport synchronization and radar or Einstein synchronization schemes are being compared (Section 1.5.3). They test any alternative to a metric theory in which the equivalence of these synchronization techniques is not assured, so that the apparent speeds of light are no longer equal (Section 2.1.3). Experimental confirmation of any inequality would be evidence for the inadequacy of at least some of the metric-related hypotheses of general relativity. This, in principle minor, clarification exposes the conventional element (as does the second probe of the voltmeter in our illustration), makes the issues independent of convention and is adequate to give formal grounding of these experimental tests. It still lacks a physical grounding in the absence of physical models which incorporate the inequality of the apparent speeds of light, the nature of the physics so tested remains obscure (Section 3.4).

Such emphases on one-way speeds have been misleading. They ignore many well-documented refutations of attempts to conduct one-way or first-order tests of relativity, e.g., by Tyapkin [213], Karlov [94–96], Trimmer [211], Winnie [233], Stedman [200], Newburgh [155], Rodrigues [185], Cavalleri and Grøn [26], Ungar [214], Clifton [33], Bay and White [13], Horedt [86], Michel [139], Grøn [71, 70], Peres [164], and Anderson et al. [5, 6].

We discuss the nature of test theories more fully in Section 1.4.2. Golestanian et al. [69] speak of a revival of experimental tests of the special theory. As Tourrenc and Melliti [209] comment, it is important to build a new parametrized theoretical framework for the analysis of high-precision experiments, especially now that it is known [192] that gravitational tidal effects could be significant in such experiments.

1.4. Other motivations for this article

1.4.1. Gauge theories

As these problems show, the traditional neglect of this topic in standard presentations of relativity is unfortunate. (Exceptions include [32, 160].) We consider also that a proper undergraduate-level analysis of this topic gives the opportunity for a splendid tutorial example of the effects of a gravitational gauge transformation. As such, it affords significant insight at an elementary level into the overwhelmingly important gauge field theories of modern physics. Seeing through this gauge freedom and gauge-theoretic structure in general relativity in the simplest possible case is a natural introductory exercise.

This elementary insight can be gained at an operational level, with helpful insight. Winnie [233] gives a remarkably clear discussion of this, showing that all velocities are conventional, and deducing the modified form of time dilation and slow clock transport. We outline this and emphasise some conclusions.

It is however more convenient to use a tensorial formulation. While Møller [148] gives the full theory of gauge transformations in general relativity, the simple 4-tensor analysis of this question in our context is virtually unknown. We show in Section 1.5 that it is economical and unambiguous.

A simple example is doubly valuable when the full theory (see Section 3.2) is nontrivial. The covariance of a theoretical framework, and the invariance of its physical predictions, under a change of convention of coordinates lies at the heart of gauge field theory, which is a vital concept in modern physics. Establishing the formal consequences of such an elementary concept as the conventionality of the zero of voltage has been of critical concern in many hundreds, possibly thousands, of Ph.D. theses which develop applications of superconductivity, QED, electroweak theory and QCD for example. Indeed, some substantial technical literature in general relativity is devoted to such gauge choice problems [148, 60, 219, 88]. We note that the gauge freedom we discuss affects the zero of voltage (Section 2.3.3) and corresponds to the freedom in what is often referred to as the gravitational vector potential, namely the mixed space–time metric components g_{i0} (Sections 2.3, 2.3.4).

We note the comments by Wald [226]: "diffeomorphisms [mapping between various manifolds] comprise the gauge freedom of general relativity", and by Goeckeler and Schuecker [68] "usually gauge fixing, while helpful in concrete calculations, masks the general structure of the theory". In our minds, not only is structure of the theory masked, but the nature of the entities that the theory refers to is obscured as well.

For these reasons, we consider it valuable to defend in some detail the same principle of gauge or coordinate invariance in the above context. In addition, our formal analysis, in giving its deeper insights into the structures of possible test theories of relativity, has helped in developing new test theories and in the motivation and interpretation of new experimental tests.

Some ancillary issues may be mentioned. First, at a physical level, any operational procedure involved in imposing some synchronization has a different character in special relativity than in a Galilean theory. Only time-like intervals imply a natural sign for temporal ordering of the terminal events, whereas there exists a universal temporal ordering in a Galilean theory. However the covariance property is the same in principle. On one view then, interpreting any synchronization scheme as a gauge choice and so as inducing the conventionality of synchronization trivialises the bearing brought by the very different space–time structure of special relativity (see Sections 1.5.1, 2.3.2).

In Section 1.5.4 we illustrate the extent of possible gauge convention (as in [81]) and depart from a universally accepted coordinate convention in Einstein's work as mentioned earlier. The conclusions of Eddington (Section 2.1.3), of Robertson [183] and of Brown and Maia [24] – that "the Michelson–Morley experiment establishes the isotropy of the round-trip speed of light, which holds in the resting frame by hypothesis, also holds in the moving frame" – are themselves dependent on a coordinate convention on lengths. Such conventions have significantly different status to those reflecting the relativity of simultaneity. Two standard conventions here reflect the use of a standard rod in all possible orientations in space for length measurements, and also the use of a standard (slowly transported) clock at all possible positions for rate or frequency measurements. Poincaré [170] expressed the existence of a convention regarding clock rate in the definition: "...that the duration of two identical phenomena is the same; or if you prefer, that the same causes take the same time to produce the same effects". The Michelson–Morley experiment supports the simplicity of the coordinate convention on length. The absence of a Doppler effect on reception (of regularly transmitted electromagnetic pulses; Bach's Air on a G string being received in the key of G) supports the simplicity of the coordinate convention on frequency.

When these standard conventions are accepted, the only remaining convention is the major issue of this article, namely the phase of the remote clock or equivalently the (choice of anisotropy of the) one-way speed of light. The standard convention of isotropic one-way light speed is in a sense less compelling than either the rod length or clock rate convention within special relativity. This measure of compulsion reflects a variety of aspects: interesting questions about the causal structure of the world, the spreading of time through space, and the mixing of temporal and spatial considerations rather than the separate definition of either. In detail, it reflects the significance of the finite speed of light in allowing a finite flexibility of choice of the anisotropy parameter κ consistent with global causality (all receptions at a local time later than that of transmission). Gauge freedom alone, then, is not the only reason for our discussing the relativity of simultaneity. Authors such as Robertson [183], Brown and Maia [24] and Mansouri [136], in contrast to Section 1.5.4, take the the rod length and clock rate conventions as axiomatic in preference to the convention over one-way light speed. Indeed, this is uniformly done when transferring between inertial frames, to reduce the formal complexity and minimise an otherwise bewildering variety of gauge choices.

Comments on the relationship between gauge transformations and relativistic test-theories, and particularly general covariance, are deferred to Section 2.5.1.

1.4.2. Test theories

One area in which synchrony issues have enjoyed much consideration, but have not however enjoyed correct usage, is in the experimental testing of special relativity. The currently favoured approach is to use a "test-theory", by which is meant a theoretical framework which contains a continuum of theories, in which a particular set of parameter values specify a theory to be tested; all other parameter combinations give rise to alternative (rival) theories. How much one theory differs from another is gauged by the difference in respective parameter values: if the parameters are chosen to correspond to physical observables, then different aspects of a theory can be investigated independently of each other. An experimental bound on the parameters, to within a given tolerance, constraints the viable test theories to a very limited if continuous subset of the full family of theories. The test-theory approach thus handles all possible theories of a given type simultaneously.

Although the theory of special relativity was formulated before the theory of general relativity, and is assumed within the latter theory to be valid in the limit of negligible gravitation, the experimental testing of special relativity with test-theories is not as extensive as in the situation of general relativity, as reviews by Damour [40] and Will [232] indicate, and none of the corresponding test theories enjoy the same status as (for example) the PPN [143, 231] or TH$\varepsilon\mu$ test theories [64] used in discussing general relativity (which both satisfy local Lorentz invariance). More emphasis is placed on dynamics in general relativity than in special relativity: the most popular test-theory of special relativity, the Mansouri–Sexl test-theory [133] concentrates on kinematical considerations and the structure of space–time. However, this bias in favour of the kinematics and space–time structure reflects the importance of both of these properties in the foundational aspects of special relativity. We attach little significance to this, since we attribute to apparently kinematic test theories dynamical presuppositions [23].

Synchrony issues have been as contentious in test-theories which violate some assumption of relativity as they are in special relativity, because the interpretations drawn depend critically on the interpretation used for the conventionality or otherwise of potentially anisotropic parameters. Mansouri and Sexl [133] and others have claimed that although the one-way speed of light is not measurable from within a purely special relativistic framework, it becomes measurable when considered within the more general context of a test-theory. Such claims are discussed in Section 3.2 where it is shown that conventionality has not been adequately handled by Mansouri and Sexl [133], and furthermore that such supposedly empirically accessible parameters within their theory have a degree of conventionality. This is shown by reinterpreting and generalising the Mansouri–Sexl test-theory to arbitrary synchrony, and demonstrating covariance for all theories in this formalism. Section 3.3 briefly reviews some experimental tests and their limitations regarding conventional quantities that they have been claimed to measure. These limitations have been obscured in many analyses of experiments because authors have failed to give proper consideration to synchrony. Section 3.3 discusses the analysis and interpretation of experiments in which synchrony-dependent parameters are evaluated. As examples, two experiments (the two-photon absorption [180] and maser phase [110]) are analysed within our recasting of the Mansouri and Sexl test-theory given in Section 3.2.

1.4.3. Test theory for general noninertial observer

Section 4.1 looks at synchronization for non-inertial observers, an aspect which has not been greatly considered in the literature. In Section 4.1 the coordinate system of an accelerated observer with arbitrary synchrony is developed, not only for the case of special relativity, but also for an arbitrarily curved manifold in a general relativistic theory. Introducing arbitrary synchrony for a noninertial observer requires a different prescription from that traditionally used for the observer who sets up a locally Lorentz, locally Einstein synchronised set of coordinates. Accordingly, the assignment of local coordinates, tetrad propagation within the observer's frame, and finally the observer's metric are discussed, and Section 5.2.1 modifies some of the assumptions in Section 4.1.

The outcome of this is a novel test-theory of local Lorentz invariance, taking into account arbitrary synchrony, reported by Vetharaniam et al. [224]. Because it refers to a test theory, it is not assumed for all members of the family of theories that the interval is invariant even under a transformation between inertial frames. As such, the test theory forms a more radical departure from general relativity than, say, the well-established PPN test theory, in each of whose component theories local Lorentz invariance is maintained. Naturally, our test theory is more vulnerable to experimental tests. This is deliberate, and we illustrate in Section 5.4 how an experiment in ring interferometry which has not yet reached the sensitivity required to bound the PPN parameters (in comparison with a proposal by Scully et al. [192]) may still be used to bound the parameters of this theory.

1.5. Four-vector formulation of synchrony change

We review the four-vector formulation of special relativistic mechanics. For example, a boost in the x direction has the form:

$$x' = \gamma(x - vt), \qquad t' = \gamma(t - vx/c^2), \qquad y' = y, \qquad z' = z, \tag{1}$$

where $\gamma = (1 - \boldsymbol{v}^2/c^2)^{-1/2}$, and $\boldsymbol{v}$ is the velocity associated with the frame change.

We rewrite these Lorentz transforms in the form $X'^{\mu} = L^{\mu}{}_{\nu} X^{\nu}$ where

$$X^{\mu} = (ct, \boldsymbol{x})^{\mathrm{T}} \tag{2}$$

(the superscript denoting the transpose) and the metric is taken to be $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. This corresponds to a matrix

$$(L^{\mu}{}_{\nu}) = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

The associated covariant components $X_{\mu} = \eta_{\mu\nu} X^{\nu}$ are boosted by the matrix $L_{\mu}{}^{\nu} = (L^{-1})^{\nu}{}_{\mu}$. The invariant interval is:

$$\mathrm{d}s^2 = \mathrm{d}X^{\mu} \eta_{\mu\nu}\, \mathrm{d}X^{\nu} = -c^2\, \mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2. \tag{4}$$

Tensor analysis makes it natural to think of this as a "distance" which is a geometrical quantity, preserved under the frame change.

The standard development of 4-vector relativistic mechanics derives the associated 4-vectors $V^{\mu} = \mathrm{d}X^{\mu}/\mathrm{d}\tau$ where $\mathrm{d}\tau$ is the proper time interval, so that $\mathrm{d}t = \gamma\, \mathrm{d}\tau$ and $V^{\mu} = \gamma(c, \boldsymbol{v})^{\mathrm{T}}$. The 4-momentum is $P^{\mu} = m_0 V^{\mu}$ where m_0 is the rest mass, so that $P^{\mu} = (mc, \boldsymbol{p})^{\mathrm{T}}$. The 4-force is $F^{\mu} = \mathrm{d}P^{\mu}/\mathrm{d}\tau = \gamma(c\, \mathrm{d}m/\mathrm{d}t, \boldsymbol{f})$.

In all of this development so far we have assumed that our observer S is using Einstein synchronization; i.e., he has an isotropic convention for light speed.

1.5.1. Nonconventional synchrony

We now propose a new set of coordinates, associated with an observer $\tilde{S}$ say, of the form [4]

$$\tilde{t} = t - \kappa x/c, \qquad \tilde{x} = x, \tag{5}$$

or

$$(\tilde{X}^{\mu}) = (X^{\tilde{\mu}}) = \begin{pmatrix} c\tilde{t} \\ \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = S^{\mu}{}_{\nu} X^{\nu} = \begin{pmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ct - \kappa x \\ x \\ y \\ z \end{pmatrix}. \tag{6}$$

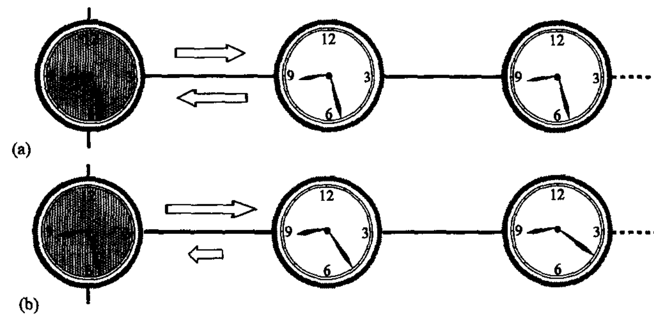


Fig. 2. (a) Einstein synchrony assumes anisotropic one-way speed of light, with clocks adjusted accordingly. (b) Anisotropic synchronization results in a re-adjustment in which a faster one-way speed from the master clock results in an earlier time setting.

Fig. 2 illustrates the relative assumptions of the standard observer S (Fig. 2(a)), and the anisotropic observer $\tilde{S}$ (Fig. 2(b)).

Note that what we have done is set up throughout the frame of a set of clocks for observer $\tilde{S}$ rivalling those used by the original observer S, at the same positions and with zero relative speed ($\tilde{x}=x$) but with a shift in the setting of each clock backwards by an amount ($\tilde{t}-t=-\kappa x/c$) which is proportional to its x displacement. We call $S^{\mu}{}_{\nu}$ a synchrony change matrix. Such a coordinate change cannot alter the physics or the predictions of the theory in any substantive way. All the consequences of this maverick assumption are discernable simply by applying this coordinate change to all relevant tensors.

The locus of a light beam in the un-tilded, "Einstein synchronization" coordinates used by S is $x=\pm ct$. This means that $c\tilde{t}=ct-\kappa x=ct(1\mp\kappa)$ and so

$$\tilde{x}=x=\pm\frac{c\tilde{t}}{1\mp\kappa}\,. \tag{7}$$

The speed of light in each direction is therefore

$$c_{\pm}=\frac{c}{1\mp\kappa}\,. \tag{8}$$

Our above example corresponded to the choice $\kappa=\frac{1}{3}$. The general result always preserves the round-trip speed as c.

Much of the philosophy literature uses an alternative (but operationally equivalent) parametrization of synchronization dating from Reichenbach (see, e.g., [189]) in which the role of κ is played by ε where

$$\varepsilon=\tfrac{1}{2}(1+\kappa)\,,\qquad \kappa=2\varepsilon-1\,, \tag{9}$$

making these speeds $c/2\varepsilon$ and $c/2(1-\varepsilon)$ respectively. Einstein synchronization is defined by $\varepsilon=\frac{1}{2}$.

At first, we might be tempted to confine our consideration to the case $|\kappa|<1$, else speeds become negative, the traveller arriving (at least on the showing of local coordinate time) before leaving. This itself is a coordinate convention; airline travellers crossing the International Date Line in an easterly direction routinely survive such treatment. Accepting any such restriction on κ has then to be done for more subtle reasons, for example ensuring that coordinates are assigned so as to ensure the formal respectability of a globally causal ordering of events, i.e., ensuring that the local coordinate times for reception always are later than those for transmission (see Sections 1.4.1, 2.3.2).

1.5.2. Velocity

Not only is the speed of light affected but all speeds are affected. The speeds of material particles are timed by their coincidences at master and slave (remote) clocks, and these remote clocks have been altered. Not only speeds are affected; so are many components of all 4-tensors, especially (but not only) those that obviously depend on speed, such as mass, force and magnetic field.

There are two ways of analysing the effects of such changes. First, one may take a particular thought experiment and track through all the times of arrival and departure in the new observer's framework so as to work out the new speeds. For example, a ball leaves the origin at $t = \tilde{t} = 0$, and arrives at $x = d$ at $t = d/v$, where v is its Einstein speed. The time shown on $\tilde{S}$'s clock is $\tilde{t} = t - \kappa d/c$, so $\tilde{S}$ should set its speed as

$$\tilde{v} = \frac{d}{\tilde{t}} = \frac{d}{t - \kappa d/c} = \frac{d}{d/v - \kappa d/c} = \frac{v}{1 - \kappa/c}, \tag{10}$$

which depends on κ. An extended and very helpful approach on these lines is given by Winnie [233].

Second, one can plunge straight into the tensorial formalism [4], recognising that the relatively general, abstract and elegant manipulations being performed amount (as may be verified) to exactly the same thing. In this case we simply identify a new (tilded) velocity and time dilation factor from

$$\tilde{V}^{\mu} = S^{\mu}{}_{\nu} V^{\nu}\,; \qquad (\tilde{V}^{\mu}) = \tilde{\gamma}(c, \tilde{\boldsymbol{v}})^{\mathrm{T}} \tag{11}$$

in complete analogy with the Einsteinian form. Since

$$S^{\mu}{}_{\nu} V^{\nu} = \gamma_v \begin{pmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix} = \gamma_v \begin{pmatrix} c - \kappa v_x \\ v_x \\ v_y \\ v_z \end{pmatrix}, \tag{12}$$

we thus identify

$$\tilde{\gamma} = \gamma(1 - \kappa v_x/c)\,, \qquad \tilde{\gamma}\tilde{v}_x = \gamma v_x\,.$$

Dividing these equations immediately and naturally gives Eq. (10). Were there any doubt about the interpretation of $\boldsymbol{v}$, then the first and longer method eliminates it.

This calculation has given another result: the gamma factor for $\tilde{S}$ has also been changed. The above equations give

$$\tilde{\gamma} = \frac{1 - \kappa v_x/c}{\sqrt{1 - v^2/c^2}}\,. \tag{13}$$

In a similar way, the time dilation factor $\mathscr{T}$ is obtained by finding the appropriate time ratio (in which the synchrony choice in the frame of the clock being monitored is irrelevant):

$$\mathscr{T} = \frac{\mathrm{d}\tilde{t}'}{\mathrm{d}\tilde{t}}_{|\mathrm{d}x'=0} = \frac{\mathrm{d}t'}{\mathrm{d}t - \kappa\,\mathrm{d}x/c_{|\mathrm{d}x'=0}} \tag{14}$$

$$= \frac{1}{\gamma(1-\kappa v_x/c)} = \frac{1}{\tilde{\gamma}}\,. \tag{15}$$

Notice that the standard time dilation factor $\sqrt{1-v^2/c^2}$ is itself a convention-dependent relation, and not a directly observable formula. This is because it is not a statement about the pattern of coincidences of events at given space–time locations, but refers to the comparison of remote events, and so is inevitably conventional. This needs to be borne in mind when "proving" it from experiment [39,61].

One can confirm this equation by the first method as above, thinking through a particular thought experiment, redefining all times. A clock moving with respect to some frame runs at a different rate to the coordinate clocks in that frame, and differently again depending on how those clocks are defined. It could even be a faster rate; $\mathscr{T}$ could be less than 1.

If the same procedure is applied to the momentum 4-vector as was applied above to the velocity 4-vector, the mass of an object, being the timelike contravariant component, acquires a conventional or synchrony-dependent element: $\tilde{m} = m - \kappa p_x/c$.

1.5.3. Slow clock transport

The anisotropic time dilation factor of Eq. (15) contains a term which is linear in v. Herein lies the full explanation of a possible answer to the question at the commencement of Section 1.2: Could we not check whether the one-way speed is isotropic or not by moving a clock slowly from the master, and seeing how it agrees or disagrees? It can agree with at most one choice of synchronization.

And that choice is the isotropic choice. All present experiments indicate to some level of precision, and all currently accepted theories predict [97] that slow clock transport synchrony agrees with Einstein synchrony.

Moving a clock slowly gives the same remote timekeeping as that given by the radar method when it is assumed that light speed is c in every direction so that $\kappa = 0$. We stress through this article that this is the proper conclusion to draw from many old and many recent experimental tests of special relativity, and in particular all the recent tests erroneously purporting to illuminate the isotropy of the one-way speed of light. Those members of a family of theories which contradict the equivalence of slow clock transport and Einstein synchrony are discredited in proportion to the magnitude of their predicted, but unobserved, discrepancy. Unfortunately, these conclusions are rarely drawn, and have been confused with a measurement of one-way speed or of the degree of its isotropy.

The equivalence of slow clock transport and Einstein synchronization might appear to prove that Einstein synchrony has been empirically determined, and any other value of κ than zero is wrong. However, any empirical result must be obtained within any member of a family of theories which are equivalent up to a coordinate transformation. Hence slow clock transport synchrony must disagree with the non-Einstein synchrony when $\kappa \neq 0$, if it is to agree with Einstein synchrony (when $\kappa = 0$). And that is indeed to be expected, thanks to the new time dilation factor (Fig. 3). If we take the

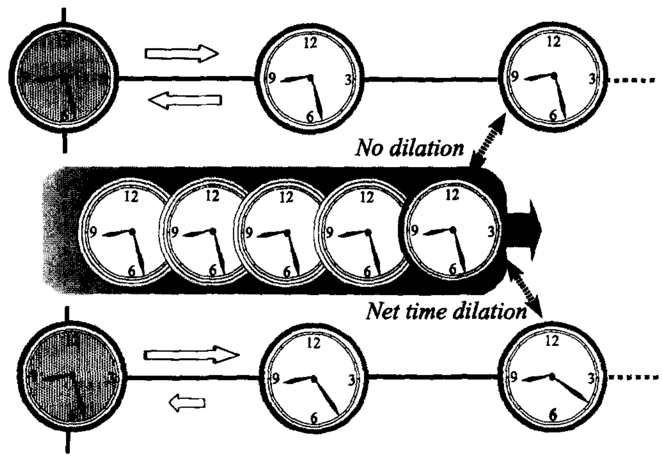


Fig. 3. Slow clock transport (middle diagram) gives a remote time setting which apparently agrees with Einstein synchrony (upper diagram). However there is no conflict with an anisotropic one-way speed of light when the resulting first-order effects from time dilation on the slowly transported clock are allowed for.

assumption that the speed of light is anisotropic seriously and consistently, we necessarily arrive at the modified time dilation expression of Eq. (15). Because of the linear term this has as an inevitable consequence that a slowly transported clock inevitably suffers time dilation, and is in disagreement with that of a (tilded) coordinate clock. This is a novel if formal result; it was not part of Einsteinian special relativity. Hence the fact that a slowly transported clock disagrees with a coordinate clock with nonstandard synchrony is no proof that the clock with nonstandard synchrony is incorrectly set; the observed net time dilation is as much to be expected on the assumption that light speeds are anisotropic as on the assumption that light speeds are isotropic.

This may now be verified quantitatively and instructively. No matter how slowly a clock is transported (at speed v say), it has to cover a finite distance d say, and so takes a time $t = d/v$. In that time its rate is changed to first order by a factor $(1 - \kappa v/c)$ (see Eq. (15)), which during a time t adds up to a setting shift of $(-\kappa v/c)(d/v) = -\kappa d/c$, or the setting change between t and $\tilde{t}$. Note that v cancels in this last calculation.

We have emphasised this point because confusion over this and allied matters lies at the heart of many physicists' mistreatments of the subject. The reason why slow clock transport and Einstein synchronization coincide in a standard metric theory is examined in Section 2.1.3.

1.5.4. Dual observer and the metric

In parallel with earlier equations, we may write the covariant components (or adjoint, or contragredient transformations) as

$$\tilde{X}_\mu = S_\mu{}^\nu X_\nu \,, \tag{16}$$

$$S_\mu{}^\nu = (S^{-1})^\nu{}_\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \kappa & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{17}$$

$$\tilde{X}_\mu = \begin{pmatrix} -c\acute{t} \\ \acute{x} \\ \acute{y} \\ \acute{z} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \kappa & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -ct \\ x-\kappa ct \\ y \\ z \end{pmatrix}. \tag{18}$$

Hence the covariant components, distinguished here by an over-prime, have the same time and a different, time-dependent position. If we try to interpret these physically as the coordinates of an observer $\acute{S}$, this can only mean an observer in a different frame with a relative speed κc ($\acute{x}=0$ when $x=\kappa ct$), but (being anti-relativity!) who insists on using the original time, as though time dilation did not exist.

$\acute{S}$ (see Fig. 4) is perfectly entitled to do this. She pays for her idiosyncrasy, as does $\tilde{S}$, but in her own way; she finds that although relativity is consistent with all experimental results, everything gets more complicated and new fishhooks emerge. For example, the round-trip speed of light is anisotropic for $\acute{S}$. To see this, consider a Michelson–Morley experiment in $\acute{S}$'s frame, in which beams are sent out-and-back parallel and perpendicular to the direction of relative motion (to S). Since these beams leave and return together, the exact definition of time interval is irrelevant, and any distinction in the inferences on round-trip light speed depend only on distance measurements. An observer T in $\acute{S}$'s frame who relates her coordinates by a Lorentz transformation from S will measure the distances, and therefore the round trip light speed, to be the same. However, $\acute{S}$'s distance measurements will differ in the parallel direction since $\acute{S}$ does not allow for a length contraction effect in her transformations. It follows that $\acute{S}$ will derive different lengths, and so different round-trip light speeds for the same observed time delay in both arms, in the two arms of her Michelson–Morley apparatus. The fact that this is at odds with the standard conventions on length definition is discussed in Section 1.4.1. The equally strained relationship of this to various round-trip axioms (Section 1.1) and the manner in which some aspects of the work of Robertson suggest similarly unusual coordinatisations is briefly discussed in Section 3.1.2.

We note in passing that neither the invariance of the (isotropic) one-way speed of light nor the reciprocity of velocities (which are implied by the Lorentz transformation) hold in a formulation where $\kappa \neq 0$. An analysis of the logical development is given by Brown [22]. In the development of relativity, the first result is replaced by the "light postulate": there is a frame (and Einstein thought of it as a resting frame, even an ether frame) in which the two-way speed of light is isotropic and independent of the source. The second component is the relativity principle: dynamical laws are indifferent as to which of two frames are used for their description, provided they are related by uniform constant motion.

One can take this a stage further: because the kinetic energy relation $\boldsymbol{f}\cdot\boldsymbol{v} = \mathrm{d}(mc^2)/\mathrm{d}t$ comes from a contraction of covariant and contravariant indices (in $\tilde{F}^\mu \tilde{V}_\mu = 0$), we may conclude that these two maverick observers have to pool their results for simplicity of definition of conserved energy. S's force has to be multiplied by $\acute{S}$'s velocity, or vice versa [4].

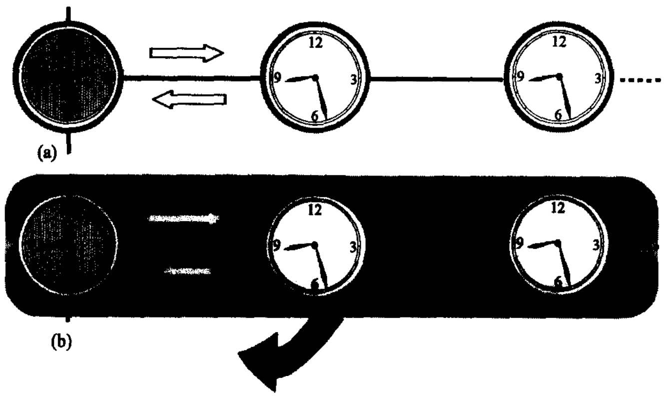


Fig. 4. A set of observers dual to the anisotropic observers are in a different inertial frame, but use the same time as in the original frame. Their derived values for mass, momentum etc. form duals to those of the anisotropic observer.

The metric can now be obtained in a variety of ways, for example:

$$\tilde{g}_{\mu\nu} = S_\mu{}^\alpha g_{\alpha\beta} S_\nu^{-1\,\beta} = \begin{pmatrix} -1 & -\kappa & 0 & 0 \\ -\kappa & 1-\kappa^2 & & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

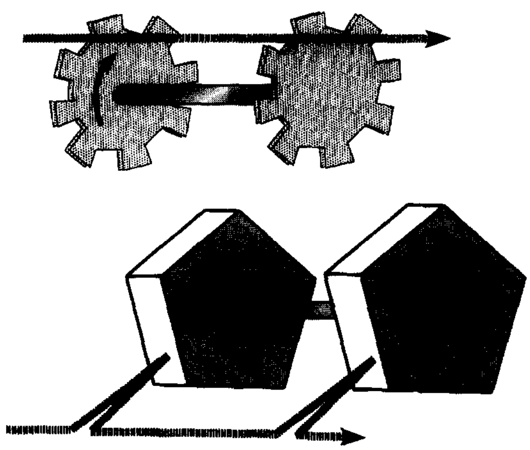
and we can check that $\tilde{X}_\mu = \tilde{g}_{\mu\nu}\tilde{X}^\nu$, etc. Gauge freedom in any theory means freedom of choice of the gauge, or metric; in this case, of κ.

This material will be generalised in Section 2.3.

1.6. One-way quests

It is natural for physicists to resort to a variety of ingenious thought experiments which purport to measure the one-way speed of light. It is a curious fact that some of the historically important measurements of the speed of light are particularly important and challenging in this connection.

Fizeau's original experiment of 1849 for measuring the speed of light involved a single toothed wheel, with the light beam passing through a gap between the cog teeth, out to a remote mirror, and then back through the same – or adjoining – gap [207]. The cogwheel, at Fizeau's father's house in Suresnes near Paris, plays the role of the master clock M of our earlier discussion in Section 1.2, and for the remote station N, Fizeau used a collimator in Montmartre. This suggests various "one-way" thought experiments, variants of Fizeau's and which as far as we know have not been actually used to estimate the speed of light, in which for example the light beam passes in one direction only through co-rotating toothed wheels. Would this measure a one-way speed? No remote timekeeping is necessary, and the light travels only in one sense (Fig. 5(a)). That this was a live question is demonstrated by Michelson's remark on Newcomb as reported in Section 1.1. Variations on this are possible and have been suggested: one can have a polygon of flat mirrors around the circumference of the rod at each end, and bounce a light beam off one if it is opportunely oriented. When the beam gets to the other end, will it arrive at such a time as to bounce off the next mirror at the same angle? (see



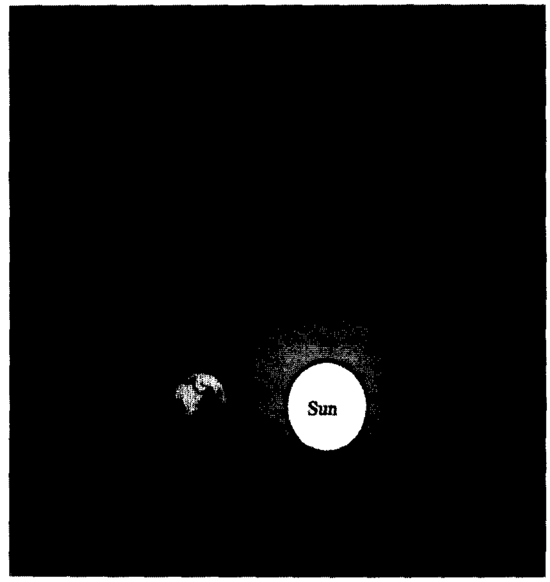


Fig. 5. A double rotating toothed wheel will allow light to traverse the system parallel to the axis, missing both sets of teeth, apparently if the one-way speed is appropriate. However, on an anisotropic theory, there is a compensating kinematic torsion in the rotation axle which allows the light to pass. The same is true for a system of rotating prismatic mirrors.

Fig. 6. Rømer's method of determining the speed of light from the apparent changes in the period of Jupiter's moons appears to be a measurement of a one-way light speed. Here one actually measures a (local) Doppler effect, insensitive to considerations of remote synchrony, and the same result is returned on a theory with anisotropic light speed.

Fig. 5(b)). The answer to that empirical question will ostensibly change with the one-way speed of light.

Another apparently irrefutably one-way determination is Rømer's method for determining the speed of light from the measured seasonal fluctuations in the apparent transit times of Jupiter's moon Io (Fig. 6). After all, no clocks and no light detectors exist on Jupiter. Incidentally, this experiment stimulated Maxwell [138] to consider the possibility of measuring the motion of the Earth through the ether; and this in turn stimulated Michelson's original interest in such matters [89].

In their seminal papers proposing a test theory for special relativity, Mansouri and Sexl (1977) insisted that this experiment measured a one-way speed for light, and declared a conventionalist account by Karlov (which we here support; see also Karlov [94–96] and Section 3.1.3) to be in error. Nearly 20 years passed before a correction to Mansouri and Sexl was published, [222,223] and in the meantime a misleading anticonventionalist approach has become customary in applications of the Mansouri–Sexl test theory.

The flaw in the one-way interpretation of this Fizeau-like experiment is again that it ignores necessary consequences and complications in the full and consistent description of a system once light speeds are deemed to be anisotropic. The slow transport clock paradox arose because extra anisotropic time dilation was ignored. The coordinate approach shows clearly another unexpected but inevitable consequence: in the Fizeau experiment, the rod must be understood to develop a longitudinal twist as its rotation rate is increased. This assertion may sound bizarre, but is a purely kinematic effect, involving no forces. It merely notes that at the same $\tilde{t}$-time (at each end) the two toothed wheels are no longer aligned once the shaft rotates at finite speed. The wheels are perfectly aligned at the same time t, regardless of rotation speed, and if t and $\tilde{t}$ agree at one end (the origin) they cannot agree at the other. So the empirical fact that the light passes between the teeth on both wheels is perfectly consistent with it going slower in the interim. The cogs appear to have been screwed out of line to compensate for the speed change, and thus to deliver the same experimental result.

As for Rømer's method, the empirical fact is the Doppler effect [11]: the moons of Jupiter appear to orbit more slowly while the Earth recedes, and faster as the Earth approaches Jupiter. The one clock in the game is Earthbound: it is a slowly transported clock. If the light speed is assumed to be anisotropic, the clock on Earth suffers a compensatory extra time dilation effect which contrives to mask the anisotropy in exactly the same manner as discussed before. Hence the observed Doppler effect is synchrony-independent. Einstein synchrony is merely the least complex of the possible theoretical frameworks for the explanation and interpretation of the experiment. In effect the measured quantity is the round-trip speed of light in the Earth's orbit. Some of the literature on this is discussed in Section 3.1.3.

Some final points may be mentioned. It may be that some other combination of one-way speeds is synchrony-independent and so measurable; attempts here include that of Nikol et al. [156] and Atzmon et al. [9], who compare neutrino and photon speeds e.g. from supernova SN1987a. In case it is wondered why special play is made in this article of the freedom in coordinatizing time rather than length, we discuss that possibility in Section 2.5.2. Some authors have for tutorial purposes taken pains to identify synchrony-independent effects [187] or equally to highlight synchrony-dependent effects by contrast [66].

2. Synchrony in special relativity

2.1. Simultaneity and synchronization

2.1.1. Early history of one-way conventionality

As discussed in the introduction, the conventionality of distant simultaneity was realised with varying clarity from the late 19th century onwards. It has a long and involved history of debate as a major point of contention in the interpretation of special relativity. The definition of simultaneity conventions, being intertwined with the concept of speed, became an issue with the advent of attempts to measure the one-way speed of light as a means to verify the existence of the aether – and thus the existence of absolute space. According to Galilean relativity, mechanical motions are insensitive to uniform motion and thus cannot be used to detect a preferred frame. Sklar [196] points out that Maxwell's reduction of light to electromagnetic radiation provided hope for the detection of absolute space: because electromagnetic waves were considered to need a medium – the aether – for propagation, an observer moving with respect to the aether (which was identified with absolute space) would detect a direction-dependent variation in the speed of light.

In 1904, and unknown to Einstein, Lorentz had presented a transformation equation for the time co-ordinate that was mathematically equivalent to that obtained by Einstein [121]. Lorentz's concept of time was that of a "local time", a concept which he had originally introduced in 1895 [120] when establishing Maxwell's equations in a frame in motion with respect to the aether frame. In this 1895 paper, Lorentz expressed the local time, t_L, for a frame moving at a speed v with respect to the aether frame, in terms of the spatial and temporal co-ordinates, x and t, of the aether frame, where $t_L = t - vx$. Later that year Lorentz used the same concept of local time, but with a transformation equation equivalent to what is now known as the time component for the Lorentz transformation. With this transformation for time, and the "Lorentz–FitzGerald" contraction factor for transforming the spatial co-ordinate, Lorentz was able to obtain the proper transformation equations for Maxwell's equations, although only for the case of electrostatics [121]. For Lorentz, t_L was merely a mathematical time co-ordinate without physical significance; the true time remained the absolute Galilean time. For Einstein, however, Lorentz's local time became the real physical time for a moving observer. Einstein made a comment on the significance of this transformation in a review article published in 1907 [49]. Noting the difficulties of Lorentz's theory he remarked [52, p. 253]:

> "Surprisingly, however, it turned out that a sufficiently sharpened conception of time was all that was needed to overcome the difficulty discussed. One had only to realise that an auxiliary quantity introduced by Lorentz and named by him "local time" could be defined as "time" in general".

Furthermore, Lorentz [122, p. 321] himself, when comparing Einstein's theory to his own, remarked in 1915 on the significance of the same change in understanding the nature of time within a co-ordinate system: "The chief cause of my failure was my clinging to the idea that the variable t [the time of the aether frame] only can be considered as the true time and that my local time t' must be regarded as no more than an auxiliary mathematical quantity". Moreover, as Zahar [234, p. 73] has pointed out, Lorentz's failure to present the correct transformation equations for moving bodies may be traced directly to his particular understanding of local time.

A number of elements of Einstein's analysis of simultaneity may also be found in essays by Poincaré. In an essay in 1898 Poincaré noted the distinction between deciding on the simultaneity of events that occurred at the same place and those that occurred at distant places [170]. Since there is no access to a universal time to order distant events, one must decide on their simultaneity or otherwise on the basis of a convention. For Poincaré, neither light synchronization nor slow clock transport provides an escape from the conventionality of simultaneity. He commented (referring to an astronomer who has used the value of the speed of light),

> "He has begun by *supposing* that light has a constant velocity, and in particular that its velocity is the same in all directions. That is a postulate without which no measurement of this velocity could be attempted. This postulate could never be verified directly by experiment ..."

In several essays, both before and after Einstein's 1905 paper (see, e.g., [168, 169]), Poincaré presented a method for synchronising clocks based on the exchange of light signals which was in many essential respects the same as that presented by Einstein. Fölsing [63, p. 176] has recently proposed that Einstein and his friend Besso probably had before them these papers of Poincaré when they met for discussions in the year prior to the formulation of the Special Theory (more details on Einstein's possible contact with Poincaré's writings may be found in the footnote annotations to Einstein's 1905 paper in Ref. [52] and the Introduction to that volume of Einstein's papers).

For Poincaré the intent was to explicate Lorentz's notion of "local time". He noted that when it was assumed that transmission times in the directions A to B and B to A were equal, then clocks in a frame moving with respect to the aether would be synchronised in a way that would show the local time at that point [168]. While one may surmise that a number of these ideas were important to Einstein's analysis of time and simultaneity, the way Poincaré deals with these issues is significantly different from the approach of Einstein (for a discussion of these differences see the texts by Miller [142], Torretti [208] and Zahar [234]). Nevertheless the discussions of Poincaré as well as those by Lorentz and Einstein in the decades surrounding the birth of special relativity show the intimate manner in which matters to do with time co-ordinates and synchronization were involved in the forging of the physical and conceptual basis of special relativity.

2.1.2. *Slowly transported clocks: Poincaré*

In his 1898 essay, Poincaré [170] mentioned the use of transported clocks to determine the time at different places on the Earth. Such transported clocks provide one of the rules for investigating simultaneity, and by this procedure the problem of simultaneity for Poincaré became one of determining the measure of time that is recorded on the clock. The latter, however, entails the comparison of different time intervals, and given the absence of direct awareness of the equality or otherwise of two different time intervals, one needs to provide a definition. While pendula and the repetition of certain phenomena, such as the rotation of the earth, provide standards for such comparisons, they do, however, implicitly involve the postulate that "the duration of two identical phenomena is the same". This is a convenient and reasonable way to define equality of intervals, but for Poincaré such a definition was not imposed by nature. Thus for Poincaré the conventional element in the use of transported clocks reduced to a more basic one of the convention in setting of clock rates.

That the clocks may be affected by movement was not mentioned by Poincaré. Einstein, however, remarked in his 1905 paper [48] that where two separated clocks are synchronised with each other using isotropic (Einstein) synchronization, there is a loss of synchronization if one of them is moved. Other than that, however, he did not discuss, in that paper or elsewhere, the procedure of using the transport of clocks for the determination of simultaneity.

Poincaré's reasons for the conventionality of clock transport are different from later reasons. For Poincaré the conventionality was to do with successive time intervals: it was a convention to say the times between successive ticks on a pendulum are equal. There is no way of placing them side by side and comparing them. Thus in his 1898 paper [170] he linked the conventionality of simultaneity via clock transport with this sort of conventionality, which differs from more recent discussions of slow clock transport synchronization.

2.1.3. *Conventionality and slowly transported clocks: Eddington*

In a text first published in 1923, Eddington [44] discussed, apparently for the first time, a procedure for synchronization using the slow transport of clocks. There was no attempt, however, as with later authors such as Ellis and Bowman [53], to avoid the circularity present in the determination of simultaneity via light signals.

To Eddington, the need for such a procedure stems from what he refers to as the "indeterminateness of the space–time frame". Frames of reference for Eddington are "fictitious" and coordinates established for frames contain an "arbitrary element analogous to an orientation". We note in passing that the language and notions implicit in this perspective are those we now embody in the concept of gauge freedom.

Eddington enunciates a "fundamental hypothesis": "Everything connected with location which enters into observational knowledge – everything we can know about the configuration of events – is contained in a relation of extension (or interval $\mathrm{d}s$) between pairs of events". The correct handling of the concept of location which Eddington is emphasising here is not so much the adoption of a relativistic perspective as the recognition that only relative distances have observational significance.

The issue of distant clock synchronization arises in a discussion which establishes an appropriate form for the interval $\mathrm{d}s$. In particular Eddington examines what happens to the reading of a clock while it is being moved from an event at position $(x_1,0,0)$ and time t_1 to another at position $(x_2,0,0)$ and time t_2.

Eddington begins by taking the differences in the clock readings at the beginning and end of the journey as being prop ortional to $\int_1^2 \mathrm{d}s/c$. If we write the interval in the form given by $\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + 2c\kappa\,\mathrm{d}x\,\mathrm{d}t + \mathrm{d}x^2$ (Eddington used $-\alpha$ for our κ), this leads to a difference in the clock reading given by

$$\Delta = \int_{t_1}^{t_2} \mathrm{d}t \left(1 - \frac{2\kappa}{c}\frac{\mathrm{d}x}{\mathrm{d}t} - \frac{1}{c^2}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2\right)^{1/2} . \tag{19}$$

In the case of a small relative velocity, this is approximately

$$\Delta = \int_{t_1}^{t_2} \mathrm{d}t \left(1 - \frac{\kappa}{c}\frac{\mathrm{d}x}{\mathrm{d}t}\right) = (t_2 - t_1) - \frac{\kappa}{c}(x_2 - x_1) . \tag{20}$$

Eddington [44, p. 15] then remarked that the clock, if moved sufficiently slowly, will record the "correct time-difference" if, and only if, $\kappa = 0$. Eddington also considered the directional dependence of this result. His reference to this procedure as one which leads to a measure of the "correct time" seems to reflect a presupposition that a slowly transported clock should agree with the coordinate clocks of the inertial frame, as well as a presupposition that whether clocks are at rest or in motion their mechanisms "record an equal interval". However, it is clear that Eddington sees his result as a "convention" and not as an argument that one must (empirically) have $\kappa = 0$. Indeed, as indicated in a following quotation, he is quite explicit about the conventional nature of this method.

Eddington is also quite explicit about the need to make assumptions about the one-way velocities of light to establish relativistic formulae, and to establish a time coordinatization throughout a frame. For example, when discussing the velocity of light he notes that the Michelson–Morley experiment revealed light to have the distinctive property of being a "fundamental velocity". It compared "there-and-back" journeys of light and showed them to be uniform for all directions. (This itself is strictly an avoidable but almost universal convention; see Section 1.5.4.) He also remarks: "When this proof is compared with the statement commonly (and correctly) made that the equality of the forward and backward velocity of light cannot be deduced from experiment, regard must be made to the context". Eddington notes that the Lorentz transformation (which he has derived) stem from the fundamental hypothesis mentioned above, and is not "a pure induction from experiment". For example, the fact that the Michelson–Morley experiment is of second-order has not been used. On the two methods, Eddington emphasized the conventional nature of both:

> "We can scarcely consider that either of these methods of comparing time at different places is an essential part of our primitive notion of time in the same way that measurement at one place by a cyclic mechanism is; therefore they are best regarded as conventional."

On an issue that has emerged as one of central importance in the test theories we later consider, viz. the equivalence between the method of synchronization of clocks by the use of light signals (with the convention of equal back and forward light speeds) and that of slow clock transport, Eddington observed:

> "Neither statement is by itself a statement of observable fact, nor does it refer to any intrinsic property of clocks or of light; it is simply an announcement of the rule by which we propose to extend fictitious time-partitions through the world. But the mutual agreement of the two statements is a fact which could be tested by observation, though owing to the obvious practical difficulties it has not been possible to verify it directly."

Thus he claimed that although these statements are conventions, they are empirically related, and are not independent. This all suggests that an empirical test of the equivalence would mean a test of the "fundamental hypothesis" and of the directional independence (and observer independence) of the round trip speed of light.

Eddington's discussion is therefore a very careful one, and one in which he is especially careful to isolate that which can be determined by experiment and that which comes from general assumptions (such as the fundamental hypothesis) and conventions. Eddington then deserves credit for recognising the equivalence of Einstein synchronization and of slow transport synchronization in a metric theory, for a careful discussion of its origins, and for noting its empirical status.

Eddington indeed deserves credit for recognising that slow transport of a standard clock on a given path is equivalent to setting coordinates so that the forward and return light journey times along a path are the same, even when the coordinatisation this gives is path-dependent. The change in reading of a standard clock at the beginning and end of a journey $1 \rightarrow 2$ is the change in proper time τ, where for any metric the infinitesimal interval $ds^2 = -c^2\,d\tau^2 = g_{00}c^2\,dt^2 + O(d\boldsymbol{x})$. Under slow transport ($d\boldsymbol{x}/dt \rightarrow 0$), then, $d\tau = \sqrt{-g_{00}}\,dt$. This interval (and proper time change) is independent of coordinate choice, and we may estimate it piece by piece using the Minkowski metric of the local Lorentz frame LLF relevant for each infinitesimal part of the journey. In this local frame, the Einstein synchronisation convention of special relativity holds, according to which coordinate time t_{LLF} at all points is established by equalising the one-way ($1 \rightarrow 2$ and $2 \rightarrow 1$) apparent flight times of light in a round-trip journey on a given path. In this convention $g_{00} = -1$, and we derive $\tau = \int dt_{\mathrm{LLF}}$. Hence slow clock transport time τ along a given path always agrees with the change in a particular global coordinatisation of time which itself is based on equalising the one-way parts of round-trip light travel times over a sequence of infinitesimal steps on that path and therefore over its full extent.

2.1.4. Slowly transported clocks: Reichenbach and others

Reichenbach [177], and later Grünbaum [73], with their ε characterization (see Eq. (9)) of simultaneity relations, promoted the conventionality of the one-way speed of light in the philosophy of science arena to a degree which identified them with the conventionality thesis itself. According to the Reichenbach–Grünbaum thesis of conventionality of simultaneity, any choice for ε between 0 and 1 (or of our κ, if its modulus is less than 1) is valid.

In his major text, *The philosophy of space and time* [177], Reichenbach discussed a number of attempts to determine "absolute simultaneity". He presented two criticisms against the use of the transport of clock procedure for attempting to establish absolute simultaneity. One problem is the dependence of clock settings on the path and speed of a transported clock. The other problem is

> "... that even if relativistic physics were wrong, and the transport of clocks could be shown to be independent of path and velocity, this type of time comparison could not change our epistemological results, because the transport of clocks can again offer nothing but a *definition* of simultaneity."

Ellis and Bowman [53] exemplify the later counter-criticism that this is a trivial type of conventionality:

> "...this only shows that distant simultaneity is conventional in the trivial sense that any quantitative equality between two things at a distance is conventional. If this is all there were to Reichenbach's conventionality thesis, it would be absurd to devote so much time to discussing it."

Bridgman mentions two ways of synchronising clocks other than the standard Einstein method using light signals. One is to use "superlight" velocities [21, p. 59]. The essence of this method is to have a search light sweep out a distant set of clocks all at a given distance from the source and to have the clocks to be set when the sweep reaches them. Reichenbach's critique [177] of this method is not decisive for Bridgman who regards this method as "definite and unique". More importantly, he notes [21, p. 61]:

> "There is no reason in logic why distant simultaneity defined in this way should not be identical with distant simultaneity as defined by Einstein. In fact, the present presumption is that the two are identical. It is ultimately a question for experiment to decide."

Thus Bridgman considered the coincidence of the two methods of determining simultaneity to be an experimental issue; furthermore, his critique of Reichenbach involved a critique of Reichenbach's notion of time and causal order.

Bridgman's other method is clock transport. He notes the inadequacy of the comments of Reichenbach and Grünbaum to the effect that because clocks are affected by motion they cannot be used for synchronization. Instead, he invoked the use, following Ives, of "self-measured" velocities which he claims are "uniquely determinable without further ado". This does not, however, provide an escape from the conventionality of simultaneity: the "self-measured" distance of the trip is affected by synchronization convention because the path traversed is not in the rest-frame of the observer.

In addressing these issues, Ellis and Bowman [53] made several claims in their 1967 paper which attracted detailed attention from authors such as Winnie [233] and Grünbaum et al. [74]. They took the position that slow clock transport and light signal procedures are logically independent of each other. They based this claim on a somewhat obscure demonstration as to how one can develop a non-standard formulation of special relativity for an arbitrary value of the synchrony parameter ε which nevertheless is supposedly consistent with the acceptance of slow clock transport synchronization. From here they concluded that slow clock transport can be used to "test empirically the principle of the constancy of the one-way velocity of light". Ellis and Bowman also regarded Rømer's method as a valid method for determining the one-way speed of light. Naturally, then, they took the position that the Reichenbach–Grünbaum thesis of the conventionality of simultaneity is false. Ellis [54] replied to the panel discussion Grünbaum et al. [74], and in a recent letter to the authors summarises his view as requiring an "epistemically isotropic" criterion for simultaneity, in which "no knowledge of direction in space is required to determine whether or not" simultaneity is satisfied. This also rules anisotropic synchronisation conventions out of consideration. However this criterion is not compulsory on a coordinatisation, which by definition requires a specific orientation. Ellis also stated correctly in paraphrasing work of McPhee that if we "insist on reciprocity of relative velocities ... then there is no leeway for [anisotropic synchronisations]". Brown [22] has discussed the nontrivial nature and interrelationship of several such reciprocity criteria; again Ellis's criterion is no more compulsory than the choice of isotropic synchronisation to which it is equivalent. More generally, one cannot enforce requirements on a simultaneity relation without begging such questions.

Winnie [233] gives a reliable and helpful discussion of slow clock transport, especially with reference to these views of Ellis and Bowman [53]. Winnie shows that slow clock transport is compatible with all synchrony choices, and thus cannot be used to distinguish any particular synchrony choice as correct. Winnie showed that numerical coincidence of the slow clock transport and Einstein synchronization could be demonstrated using a generalized Lorentz transformation. Salmon [188] commented:

> "... it follows from the ε-Lorentz transformations that standard signal synchrony must coincide with slow clock transport synchrony. From this it follows that Rømer's method does not constitute an independent method for ascertaining the one-way speed of light within the special theory. It shows that, whatever value we assign to ε, slow clock transport synchrony must agree with standard signal synchrony. Rømer's method ... constitutes a test of the factual content of special relativity..."

In this view (with which we concur) any experimental divergence between Einstein synchronization and slow clock transport would constitute an experimental violation of special relativity. Mansouri and Sexl [133] show this explicitly within their test theory, linking this to the choice of time dilation parameter.

Ellis [55] contended that Reichenbach was wrong to defend the conventionality of simultaneity, arguing that the method of light signals presents us with a circular argument. The presence of other logically independent procedures for establishing the relationship of distance simultaneity such as slow clock transport undermines his claim of circularity. Ellis acknowledged that given the method of light signals based on an isotropic one-way speed of light as well as the method of slow clock transport one still needs to make a choice between them. In such a situation, if the method of light signals is taken as conventional then the method using slow clock transport can be verified against it and becomes empirical, and vice versa. However, if it is an arbitrary choice as to which to make empirical and which conventional, then to Ellis it makes no difference to our practices or beliefs if we think of both methods as empirical. This position blurs the important and subtle logical and empirical relationship between the two methods.

Ellis and Bowman's paper is however significant for the issue of the transitivity of synchronization, the key ingredient for a simultaneity relationship to be an equivalence relationship. They showed that with constant round-trip speed of a light for a path along an n-sided polygon of successive lengths a_1, $a_2, a_3, \ldots, a_n$ with the corresponding one-way speeds of light along each length determined (in the notation of this article) by $\kappa_1, \kappa_2, \kappa_3$ the vertices synchronized using such light signals is preserved if we have the relationship:

$$\sum_{i=1}^{n} a_i \kappa_i = 0 . \tag{21}$$

Only then, indeed, is the empirical result, foundational for relativity, satisfied that the polygonal (as well as linear) round trip speed of light is c (Section 1.1). This condition can be seen as generalizing an early result of Reichenbach, which demonstrated that Einstein synchronization preserved the transitivity of synchronization for points at the vertices of a triangular path [178, Section 11]. This result of Ellis and Bowman was disputed by Grünbaum [74] on the grounds that they had imposed an unnecessary restriction on procedures for synchronizing pairs of clocks. This line of argument is consistent with his notion of "topological simultaneity", which does not hold that simultaneity relationships form an equivalence relation (for a discussion of topological simultaneity see Section 2.2.1). However, the result of Ellis and Bowman demonstrates how nonstandard synchronizations can be established throughout a frame even with a directional dependence of $\boldsymbol{\kappa}$, if constrained to ensure transitivity of the synchronization. This is automatically satisfied if the round-trip axiom of Section 1.1 is satisfied (see Section 2.3.4).

Friedman [62] also argued an anticonventionalist position, viewing the slow clock transport method as allowing a determination of standard synchrony without a vicious circularity. He regarded slow clock transport as exploiting a connection between Einstein synchronization and proper time, and thus illustrating the manner in which Einstein synchronization is "deeply embedded in relativity theory". Moreover, he claimed [62, p. 317] that "one cannot question the objectivity of this relation without also questioning significant parts of the rest of the theory". Friedman here had in mind the coincidence of slow clock transport and Einstein synchronization. To him this was a very important result in that the formalism of special relativity, even when generalised in synchrony, leads to a conclusion about Einstein synchronization [62]:

> "In particular, one cannot maintain that distant simultaneity is conventional without also maintaining that such basic quantities as the proper time metric are conventional as well."

It appears that Friedman's reading of Winnie's result was quite different from that of Salmon's as quoted above. All this is part of a general philosophical position argued for by Friedman. He maintained that a "good theoretical" structure is that which is connected to other parts of the theory. The theoretical structure forms an edifice, and if the parts are connected, then testing one means a test for the rest.

Friedman thus argued against geometrical conventionalism. If one allows the possibility of different spatial geometries and takes the position that they are all equally possible, then one has to introduce some universal quantity to account for the form of some of those geometries. But to Friedman this move is bad; the extra quantity has no explanatory significance and it only is used to allow one to entertain different spatial geometries. A similar move in the Newtonian context would be to postulate an absolute reference frame, and to introduce a speed parameter V which labels the speed of each frame with respect to the absolute frame. But V is arbitrary and disconnected from any other feature of Newtonian physics. Like ε, it is a "bad" parameter: it is not related to any other part of the theory.

There is a hint of this view in the comment by Torretti [208]:

> "Reichenbach's rule, as normally understood, does no more than expand it [the simultaneity relation] to a six-parameter family by the cheap expedient of associating every inertial frame with the *full* three-parameter family of simultaneity relations adapted to each."

Presumably Torretti means by "cheap expediency" that no more physical insight is obtained by the introduction of Reichenbach's ε. Such a position is inadequate for the following reasons. First, it does not allow that it is a physical feature of the world that allows the introduction of ε. Second, many quantities (such as potentials and phases) in physical theories fail to have numerical values determined by the empirical situation but yet are very significant features of these theories. Third, showing that results are synchrony-independent is by no means a trivial exercise; the history of this issue has shown the importance of carefully separating out the dependence of results on the choice of synchronization.

The details of the operational argument underpinning the conventionalist thesis have been discussed at length by Winnie [233], who discussed the consequences of various synchronization schemes on measurements of relative velocities, showing that they, along with time dilation and length contraction effects on a one-way trip, are conventional in nature (see Section 2.5.1).

Jammer [89] takes a conventionalist line in such statements as "one of the most fundamental ideas underlying the conceptual edifice of relativity, as repeatedly stressed by Hans Reichenbach and Adolf Grünbaum, is the conventionality ingredience of intrasystemic distant simultaneity".

Selleri [195] seems to miss some of the essential points discussed above when he states that "Only the famous experiments on the occultation of Jupiter's satellites (Römer, 1676) and on the aberration of light (Bradley, 1728) were one-way measurements, even if not very precise ones... Most contemporary authors ... seem to believe that the one-way velocity of light is not measurable *as a matter of principle*. It is fortunately still possible to disagree, and to believe instead that, owing to serious practical difficulties, it has never been measured up to the present time". Elsewhere Selleri [194] endorses what he calls "Nature's choice of synchronization", by which he means the Mansouri–Sexl case of absolute simultaneity (when one chooses an anisotropy in one frame so as to remove any space dependence in the time transformation: $t' = \gamma t$). This rests on his suggestion that only with such a choice is the Sagnac effect explicable in the rotating frame. However in patching local Lorentz transformations he omits the global asynchronization effect in a rotating frame (itself a logical foundation of the Sagnac effect). In another article, Selleri [193] inconsistently assumes the standard time-dilation relation (as opposed to Eq. (15)) while attempting to discuss a more general synchronization.

2.2. The Robb–Malament construction

2.2.1. Grünbaum, Robb and Malament

Within the community of philosophers of science most critiques of the Reichenbach–Grünbaum thesis of the conventionality in recent years have drawn on a 1977 paper by Malament [131], also the much earlier work of Robb [182] on the causal structure of the space–time of special relativity to which Malament drew attention. Malament's intent was to show how the causal structure of space–time (as derived from the light cone structure) itself enabled Einstein simultaneity to be defined, and moreover, that even with, as Malament phrased it, "seemingly innocuous" extra conditions (of "implicit definability"), Einstein simultaneity was the only possible simultaneity that could be derived from such structure. However the claims made about this notion of "implicit definability" and the significance of Malament [131] have been excessive.

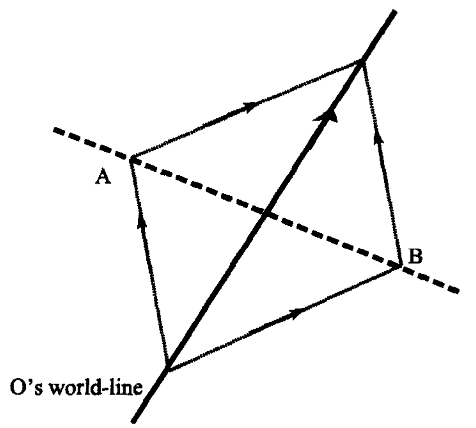
Malament [131] was concerned to counter a claim of Grünbaum [73] that the simultaneity relationship between events is not uniquely definable in terms of the causal structure of space–time and for this reason alone must be conventional. Grünbaum had specified a notion of "topological simultaneity" in terms of the causal structure of spacetime, by defining two events to be topologically simultaneous if each is outside the other's light cone. (Redhead perceptively notes that this relationship would be better referred to as absolute simultaneity [175].) It is clear, as Grünbaum indicates, that such a notion does not specify simultaneity as an equivalence relationship, because the set of pairs of events that are topologically simultaneous do not form an equivalence relationship. The finiteness of the speed of the fastest causal structure determines these features of the relationship of topological simultaneity. This relationship does not provide a metrical definition of simultaneity in the manner of Section 1.5.4 (with definitions involving a choice of ε or κ). Indeed, the indeterminate nature of the relationship of topological simultaneity shows the need for a further conventional step to specify a metrical relationship of simultaneity. Grünbaum succinctly states his position as follows [73, pp. 29–30]:

> "When I say that metrical simultaneity is not wholly factual but contains a conventional ingredient, what am I asserting? I am claiming none other than that the residual non-uniqueness of logical gap cannot be removed by an appeal to facts but only by a conventional choice of a unique pair of events at P and at Q as *metrically* simultaneous from within the class of pairs of events that are topologically simultaneous."

Malament [131] begins with a demonstration of the straightforward result that the orthogonality of a hypersurface of simultaneity to the world-line O, defining rest within an inertial frame, is equivalent to Einstein synchronization in that frame. Then using a parallelogram construction of Robb [182] in 1914 Malament indicated how orthogonality can be defined in terms of a parallelogram formed of null rays (light paths) whose diagonals form the world-line, O, and the hypersurface of simultaneity.

Such a parallelogram describes the history of a sequence of events in which two light rays (the dotted lines in Fig. 7) are emitted in opposite directions from the origin O of some inertial frame S, each then triggering a returning ray which arrives at that origin with its fellow. It follows from the manner of construction of the Robb parallelogram that the two opposite corners A, B of the parallelogram, which mark the events of emission of the returning rays, necessarily stand in the relation of Einstein synchronization to each other within S. Further details of this result may be found in [123, 175].

Redhead's exposition clearly shows how Minkowski orthogonality of a timelike and spacelike line (the result that the product of their gradients is $1/c^2$) is invariably satisfied by the diagonals of a parallelogram of world-lines in which adjacent sides have opposite slope. The fact that adjacent null rays have equal but opposite slopes in the Robb parallelogram ensures that Einstein synchronization with its unique symmetry can be defined directly. All of this may be discussed without introducing coordinates. (The uniqueness of this result is analogous to the way in which the definitions of a rhombus and its diagonals enable a 90° angle to be defined without further nonmetrical specifications.) Thus there is a manner of specifying Einstein simultaneity uniquely in



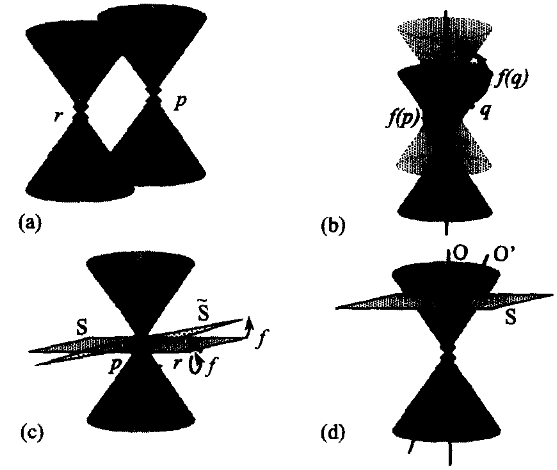


Fig. 7. The Robb–Malament parallelogram construction for a purely geometrical definition of Einstein synchrony. O emits two light beams which reach A, B respectively and are reflected to O. If they leave O together and arrive at O together, their reflection times at A, B stand in the relation of Einstein synchronization for O (but not for an observer in a different inertial frame).

Fig. 8. (a) Grünbaum's definition of topological simultaneity: two events p, r outside each other's light cone. (b) Causal automorphisms f preserve the causal connectibility of events. (c) A synchrony transformation $S \to \tilde{S}$ changes from a hyperplane of simultaneity which is orthogonal to the origin world-line to one which is nonorthogonal. (d) A change of inertial frame, from origin world-line O to O', renders an Einstein synchrony a non-Einstein synchrony (the hyperplane S is orthogonal to only one world-line).

terms of causal relationships. This result alone does not confer any compelling status to Einstein synchronization, and therefore does not challenge the essential point of the conventionalist thesis.

2.2.2. *Implicit definability*

Malament [131] proceeds to make a much stronger claim than this definability of Einstein synchronization in terms of the causal structure. He shows that Einstein synchronization is the *only* simultaneity relationship which satisfies "seemingly innocuous" extra conditions; only Einstein synchrony is "implicitly definable" from the structure of a world-line, O, together with the causal structure (the causal connectibility of events).

Malament's notion of implicit definability requires the concept of causal automorphisms. These are mappings of (coordinatized) events which preserve the relationship of causal connectibility between events (Fig. 8(b); contrast Grünbaum's definition of topological simultaneity in Fig. 8(a)). Thus, if p and q are able to be causally connected (i.e., one of these is in the light cone of the other, or equivalently the interval is timelike), then a mapping f is defined to be a causal automorphism if and only if for all such pairs p and q, f preserves this relationship, i.e., $f(p)$ and $f(q)$ are causally connectible.

In addition, Malament [131] defines an O-causal automorphism as such a mapping which possesses the additional property that the world-line O is mapped to itself, i.e., that p is on O iff $f(p)$ is on O.

A simultaneity relationship $S(p,r)$ (a Boolean function of a pair of events: true if p,r are simultaneous, false if not) is taken to be *implicitly definable* from the relation of causal connectibility and the world-line O, if and only if for all points p and r (now with a space-like interval) and for an O-causal automorphism $f : p \to f(p)$

$$S(p,r) \quad \text{if and only if} \quad S(f(p),f(r)) . \tag{22}$$

Malament [131] argues that demanding implicit definability for any simultaneity relationship S with respect to O, i.e., the requirement of Eq. (22) for all O-causal automorphisms f, limits the simultaneity relationship S to Sim_O, namely standard Einstein synchronization (the choice of $\varepsilon = \frac{1}{2}$ in Eq. (9)) relative to O as the observer world-line (provided the other conditions hold, namely S is an equivalence relationship, and nontrivial in the sense that there exist points $p \in O$ and $q \notin O$ with $S(p,q)$).

This argument has been widely accepted. Although, as described in Section 2.2.3, Redhead challenges the anticonventionalist interpretation of Malament's result he remarks: "Malament (1977) has proven the remarkable result that standard synchrony is the only nontrivial equivalence relation even implicitly definable from the relation of causal connectibility and the world-line of the origin of an inertial frame of reference" ([175, p. 114]). Torretti [208] accepts that Malament's discussion (as also an earlier related discussion by Zeeman [237]) of implicit definability and causal automorphisms limits consideration to the standard Lorentz transformations and to hypersurfaces of simultaneity orthogonal to the world-line: "More importantly perhaps, Malament proved that simultaneity by standard synchronization in an inertial frame F is the *only* non-universal equivalence between events at different points of F that is definable ("in any sense of 'definable' no matter how weak") in terms of causal connectibility alone, for a given F". Friedman [62] and Norton [159] give the same significance to this aspect of Malament [131].

Such causal automorphisms inevitably ensure that all pairs of points on an orthogonal hyperplane are mapped to another orthogonal hyperplane, and set up the notion of implicit definability in such a manner that it can be discussed only within the context of Einstein synchronization, reducing the substantive point to a near-tautology. By definition, Sim_O-preserving causal automorphisms preserve the light cone structure which has a symmetrical coordinatization around the world-line O. Einstein synchronization is associated with a symmetrical causal structure. It is possible that the attention which Malament's paper [131] has attracted from commentators has stemmed from a tendency to visualize a light cone as a symmetrical structure in space–time prior to considerations of coordinatization (fostered no doubt by a long textbook tradition to so represent light cones!), and from there to ascribe to its proposed symmetry about a world-line the degree of reality one gives to an upright cone in three-dimensional space. For a similar assessment to ours, if on this point only, see [37, footnote 3], also [93]. These authors note that an ontological interpretation of light cones is an essential feature of a favourable interpretation of Malament; one must be a *spacetime* realist to envision an actual shape of a light cone in an entity, "space–time," prior to considerations as to how space and time are to be put together; however different simultaneity relations may be argued effectively to modify the geometrical structure of the light cone.

There is a technical problem with Malament's proof. Suppose with Malament that we have a simultaneity relation $S(p,q)$ obeyed by $p \in O$ and $q \notin O$, and that the hypersurface Q is the set of events r such that there exists an O-causal automorphism f satisfying $f(p) = p$, $f(q) = r$. Malament argues that when $\mathrm{Sim}_O(p,q)$ does not obtain, Q is a double cone. However, as Malament notes, $S(p,q)$ implies $S(f(p),f(q))$ and so $S(p,r)$. Hence Q is the hyperplane of S-simultaneity which passes through p, and the mapping $q \rightarrow r$ is confined to this hyperplane. It may be canted with respect to O; for example, S may represent an anisotropic synchronisation of the type we consider in this article. Malament then considers any event v on O and introduces another O-causal automorphism (which Malament also called f, but which to avoid confusion we call h), chosen to give $h(p)=v$. He states that $h(Q)$ is a double cone; as before it may be a hyperplane parallel to Q. (Apparently his reason for this is to do with an implied restriction of the concept of O-causal automorphism to the inclusion of rotations and reflections.) At this point Malament assumes an intersection point w for Q, $h(Q)$ (as a defining characteristic of non-Einstein synchronisation Sim_O), and argues that w is simultaneous with all of p,q,r, etc., so that the simultaneity relation being respected can only be trivial. However Malament's construction fails here; this intersection point w for Q and $h(Q)$ will not exist if $h(Q)$ is parallel to Q. Arbitrarily restricting the concept of an O-causal automorphism to avoid this problem connotes the symmetrised double-cone structure which is mandatory for Malament's proof.

Too little geometric structure has been included in Malament's proof to point to a synchronisation scheme with a value of κ in Eq. (5) other than zero. The input is too sparse and too symmetric to support any such alternative result. Norton [159] (correctly in our view) notes the intrinsic impossibility of one world-line and its light cone structure picking out any other preferred spatial direction. Norton takes this (as we argue, incorrectly) to mean that only those structures which are symmetrically placed with respect to the world-line are consistent with the causal structures. In this, Norton has touched on one move at the heart of these attempts to ground Minkowski orthogonality on the causal structure, and from thence to argue that nature prefers a particular metrical definition of simultaneity. This leap from the absence of any preferred direction in the formulation of a causal structure of space–time to the assertion of a symmetrical structure such as that represented by a hyperplane of simultaneity orthogonal to the world-line is not justified, either logically or physically.

The Robb construction itself already demonstrated that Einstein synchronisation can be defined solely in terms of the relation of causal connectibility. Robb's analysis does not suffice to make Einstein synchronisation unique in this, as Malament's criticism of Grünbaum was aimed to demonstrate, and the above-mentioned problems in Malament's proofs and aims show that his analysis has not succeeded in building on Robb's.

In fact, alternative synchronisations such as Eq. (5) may also be defined in terms of relationships based on causal connectibility. For example, one may introduce a second world-line R into the above argument from causal connectibility, with as much justification as the introduction of the original world-line O. Then Sim_R, or Einstein synchronisation for R, can be taken to define the synchronisation scheme S_O for the observers O. This is a non-standard synchrony scheme because it is related to Sim_O by Eq. (5), κ being defined by the relative speed of O and R. (More generally we may use $\boldsymbol{\kappa}$ in Eq. (23) to parametrise the direction as well as the magnitude of this relative velocity $\boldsymbol{v}$ of O and R: $\boldsymbol{\kappa}=\boldsymbol{v}/c$.) Such a procedure has the geometric input necessary to permit the indication of a unique and preferred direction in space. Janis [90] has given a related argument, emphasising the role of such a second world-line in relating Einstein synchrony to an anisotropic counterpart, and a number of authors [4, 90, 174, 5] have already employed this link between a synchrony change and a frame change. (We note that Spirtes [198], who locates the type of conventionality at issue in simultaneity considerations as the choice of a temporal orientation in space–time rather than on the choice of any particular numerical value of ε, regards Malament's results to be extendable in a similar direction: "if an asymmetric causal connectability relation is intrinsic, then not only is standard simultaneity not conventional, there are also an infinitude of non-conventional nonstandard simultaneity relations".) There is no need to discuss "seemingly innocuous" constraints or to introduce concepts such as implicit definability; the above construction is precise and unique. Therefore Einstein synchronisation is not the only simultaneity criterion that can be specified in terms of considerations of causal connectibility. Also Redhead [175] has noted that the use of a nonorthogonal hyperplane for simultaneity still satisfies the requirements of an equivalence relationship.

Alternatively, as we specialise the allowable causal automorphisms further than Malament's specialisation to O-causal automorphisms, we can choose at the same time to broaden the class of synchronisation schemes which are respected by such transformations from the previously unique Einstein synchronisation to all of the anisotropic schemes of Eqs. (5) and (23). We may restrict attention to those O-causal automorphisms which preserve the orientation of the hyperplane of simultaneity $S(p,r)$. One may use coordinate language (as does Malament [131]) to specify this for non-Einstein simultaneity as the combination of an overall scale transformation $(t,x) \to (bt,bx)$ and to time translations $t \to t'$.

For these reasons it is prejudicial to require of any simultaneity relationship that it be preserved under all O-causal automorphisms as opposed to some other equally geometric criterion. There is nothing unique in this context about Einstein synchrony. It can indeed be specified uniquely by the choice of an appropriate geometric criterion (à la Robb), but exactly the same is true of the rivals we consider. Hence no compelling reason for Einstein synchronization emerges from such a discussion.

2.2.3. Frame change and synchrony change

That Malament's paper does not significantly aid the anticonventionalist case has been argued elsewhere. Redhead [175], following Janis [90], points out that Malament's prescription for singling out Einstein synchronization does not make it compulsory. Debs and Redhead [176] express this as follows:

> "...The conventionality thesis can be defended on the grounds that any method that establishes standard synchrony in a moving frame will automatically define nonstandard synchrony in a stationary frame, so the conventional element is restored in specifying simultaneity in the stationary frame, viz., the choice of whether to import into *that* frame the standard synchrony defined in any of the moving frames."

In particular, the orthogonality condition does not survive transformation to another choice of inertial frame. The world-line of the origin in the Robb–Malament construction singles out a preferred frame for this choice of synchronization. But this choice itself is a nonconventional and anisotropic synchronization of precisely the type of Eq. (5) for any other choice of inertial frame (Fig. 8(c), (d)). This justifies the conventionalist position in the case $|\kappa| < 1$. The arguments of Redhead and Debs are thus only a subset of those which preserve the conventionalist position, although they are perfectly adequate for this purpose.

No factual or compelling status is conferred on Einstein synchronization by this discussion, and the absence of such a status, the essential point of the conventionalist thesis, is not affected.

Causal relations neither involve coordinatizations nor are associated with a preferred spatial direction.

2.3. Generalised synchrony as vector field

Following Anderson and Stedman [4] again we extend the analysis of Section 1.5 to consider an observer who uses the coordinates

$$\tilde{t} = t - \boldsymbol{\kappa} \cdot \boldsymbol{x}/c\,, \qquad \tilde{\boldsymbol{x}} = \boldsymbol{x}\,, \tag{23}$$

i.e. who assumes that the velocity of light in the direction $\hat{\boldsymbol{n}}$ is of the form

$$c(\hat{\boldsymbol{n}}) = \frac{c\hat{\boldsymbol{n}}}{1 - \boldsymbol{\kappa} \cdot \hat{\boldsymbol{n}}}\,. \tag{24}$$

The three-vector $\boldsymbol{\kappa}$ is a vector field which defines the synchronization scheme, and is arbitrary. Exactly as in the simpler case of Section 1.5 (where $\boldsymbol{\kappa}$ is parallel to the x axis) the kinematics of this observer may be obtained by applying a synchrony transformation tensor (dependent on $\boldsymbol{\kappa}$) to the quantities corresponding to the Einstein synchronization case $\boldsymbol{\kappa} = \boldsymbol{0}$; the manner in which various quantities vary with synchrony is then readily available.

Such equations are readily derived when $\boldsymbol{\kappa}$ is a constant vector field. More generally again, κ may be an arbitrary function of position in space [4]. This extension is helpful to show the synchrony independence of standard results in the context of ring interferometry (see Section 2.3.4). And none of these choices are as general a gauge change as those considered by Havas [81]. However, the above choice of a constant $\boldsymbol{\kappa}$ is sufficient for the generalization of this subsection.

The synchrony transformation matrix, expressing synchrony dependent quantities in terms of their Einstein synchronization form, yields all necessary synchrony information and enshrines the power of the tensorial approach. The same considerations which guarantee covariance in Einstein synchronization also guarantee it in more general synchrony. Any tensor generalises under arbitrary synchrony with its behaviour under parity preserved. In this coordinatization the transformation takes the form $(S_\kappa)^\mu{}_\nu = \delta^\mu{}_\nu + \eta^{\mu 0}\kappa_\nu$, where $(\kappa_\mu) = (0, \boldsymbol{\kappa})^{\mathrm{T}}$, or

$$(S^\mu{}_\nu) = \begin{pmatrix} 1 & -\kappa_1 & -\kappa_2 & -\kappa_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{25}$$

The generalization of the Einstein synchronization components of an arbitrary tensor $T^{\mu\cdots}_{\nu\ldots}$ is related by $\tilde{T}^{\mu\cdots}_{\nu\ldots} = (S_\kappa)^\mu{}_\psi (S_{-\kappa})^\phi_\nu T^{\psi\cdots}{}_{\phi\ldots}$. In any frame with synchrony $\boldsymbol{\kappa}$, the contravariant components of a four-vector in a more general synchrony, $\tilde{V}^\mu$, are given with respect to the Einstein synchronization components, V^μ, by $\tilde{V}^0 = V^0 - \boldsymbol{\kappa} \cdot \boldsymbol{V}$, $\tilde{V}^i = V^i$, while the covariant components are expressible as $\tilde{V}_0 = V_0 = -V^0$, $\tilde{\boldsymbol{V}} = \boldsymbol{V} + \boldsymbol{\kappa} V_0 = \boldsymbol{V} - \boldsymbol{\kappa} V^0$.

Hence our earlier Eq. (10), Section 1.5.4 for the velocity and the metric become

$$\tilde{\boldsymbol{v}} = \frac{\boldsymbol{v}}{1 - \boldsymbol{\kappa}\cdot\boldsymbol{v}/c}, \tag{26}$$

$$(\tilde{g}_{\mu\nu}) = \begin{pmatrix} -1 & -\kappa_1 & -\kappa_2 & -\kappa_3 \\ -\kappa_1 & 1-\kappa_1^2 & -\kappa_1\kappa_2 & -\kappa_1\kappa_3 \\ -\kappa_2 & -\kappa_1\kappa_2 & 1-\kappa_2^2 & -\kappa_2\kappa_3 \\ -\kappa_3 & -\kappa_1\kappa_3 & -\kappa_2\kappa_3 & 1-\kappa_3^2 \end{pmatrix}, \tag{27}$$

or $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + (\eta_{0\mu}\kappa_\nu + \eta_{0\nu}\kappa_\mu - \kappa_\mu\kappa_\nu)$. Similarly Eq. (13) becomes

$$\tilde{\gamma} = \gamma(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v}/c) \tag{28}$$

$$= \frac{1}{\sqrt{(1+\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}/c)^2 - \tilde{\boldsymbol{v}}^2/c^2}}, \tag{29}$$

$$\boldsymbol{v} = \tilde{\boldsymbol{v}}/(1 + \boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}/c), \tag{30}$$

$$\gamma = \tilde{\gamma}(1 + \boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}/c). \tag{31}$$

By examining the velocity and momentum four-vectors, one sees that the generalised acceleration and Minkowski-force four-vectors, $a = \mathrm{d}v/\mathrm{d}\tau$ and $f_M = \mathrm{d}p/\mathrm{d}\tau$, respectively ($\tau$ being the (synchrony dependent) proper time of the particle in question), are related by $f_M = m_r a$ (m_r being the invariant rest mass) under arbitrary synchrony, the four-vector a having components

$$a = \{\gamma^4[\boldsymbol{a}\cdot\boldsymbol{v}/c + (1-\boldsymbol{\kappa}\cdot\boldsymbol{v}/c)\boldsymbol{a}\cdot\boldsymbol{\kappa}], \gamma^4[\boldsymbol{a}\cdot\boldsymbol{v}/c + (1-\boldsymbol{\kappa}\cdot\boldsymbol{v}/c)\boldsymbol{a}\cdot\boldsymbol{\kappa}]\boldsymbol{v} + \gamma^2\boldsymbol{a}\}, \tag{32}$$

where the generalised three-acceleration $\boldsymbol{a} = \mathrm{d}\boldsymbol{v}/\mathrm{d}t$ and is related to the Einstein-synchronised three-acceleration $\boldsymbol{a}_0$ by

$$\boldsymbol{a} = [(1+\boldsymbol{\kappa}\cdot\boldsymbol{v}_0/c)\boldsymbol{a}_0 - \boldsymbol{\kappa}\cdot\boldsymbol{a}_0\boldsymbol{v}_0/c]/(1+\kappa\cdot\boldsymbol{v}_0/c)^3. \tag{33}$$

It is easily seen that the three-vector parts of both a and f are unaffected numerically by synchrony, as is the relativistic three-momentum. One also obtains the synchrony generalization of the relativistic version of Newton's second law as

$$\boldsymbol{f} = m_r\gamma(\boldsymbol{a} + \gamma^2[\boldsymbol{a}\cdot\boldsymbol{v}/c + (1-\boldsymbol{\kappa}\cdot\boldsymbol{v}/c)\boldsymbol{a}\cdot\boldsymbol{\kappa}])\boldsymbol{v}/c), \tag{34}$$

where $\boldsymbol{f} = \gamma^{-1}\boldsymbol{f}_M$.

Derivative operators must transform like four-vector components:

$$\tilde{\partial}_\mu = (\tilde{\partial}_0,\ \acute{\nabla});\ \tilde{\partial}_0 = \partial_0,\ \acute{\nabla} = \nabla + \boldsymbol{\kappa}\partial_0\,, \tag{35}$$

$$\tilde{\partial}^\mu = (\tilde{\partial}^0,\ \tilde{\nabla});\ \tilde{\partial}^0 = \partial^0 - \boldsymbol{\kappa}\cdot\nabla = -\partial_0 - \boldsymbol{\kappa}\cdot\nabla,\ \tilde{\nabla} = \nabla\,. \tag{36}$$

2.3.1. Generalised Lorentz transformation

The form of the Lorentz transformations under generalised synchrony choice has been discussed by several authors including Edwards [45,238], Winnie [233], Anderson and Stedman [4], and (with especial care over the status of the various postulates) Brown [22].

The modified Lorentz transformation, from one frame Σ (with synchrony vector $\boldsymbol{\kappa}$) to another frame Σ' (with synchrony vector $\boldsymbol{\kappa}'$), is given by the matrix composition (compare Anderson and Stedman [4], Giannoni [65]):

$$\tilde{L} = S_{\kappa'} L S_\kappa^{-1}\,. \tag{37}$$

In analogy with Møller's [148] result for the standard (Einstein synchronization) Lorentz transformation $L^\mu{}_\nu$ from frame S to frame S' for an arbitrary boost,

$$\begin{aligned} \mathrm{d}t' &= \gamma(\mathrm{d}t - \boldsymbol{v}\cdot\mathrm{d}\boldsymbol{x}/c^2)\,,\\ \mathrm{d}\boldsymbol{x}' &= \mathrm{d}\boldsymbol{x} + (\gamma-1)\frac{\boldsymbol{v}\cdot\mathrm{d}\boldsymbol{x}}{v^2}\boldsymbol{v} - \gamma\,\mathrm{d}t\,\boldsymbol{v}\,, \end{aligned} \tag{38}$$

if the respective choices of synchrony vectors are now $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}'$, the generalised Lorentz transformation is

$$\begin{aligned} \mathrm{d}\tilde{t}' = {} & \tilde{\gamma}[1 + \boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}/c - \boldsymbol{\kappa}'\cdot\tilde{\boldsymbol{v}}'/c]\,\mathrm{d}\tilde{t} - (\boldsymbol{\kappa}' + \tilde{\gamma}\tilde{\boldsymbol{v}}')\cdot\mathrm{d}\tilde{\boldsymbol{x}}/c\\ & -[\tilde{\gamma}(1+\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}/c) - 1]\frac{\kappa'\cdot\tilde{\boldsymbol{v}}}{\tilde{\boldsymbol{v}}^2 c}\tilde{\boldsymbol{v}}\cdot\mathrm{d}\tilde{\boldsymbol{x}} + \tilde{\gamma}\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}(\boldsymbol{\kappa}\cdot\mathrm{d}\tilde{\boldsymbol{x}})/c\,, \end{aligned} \tag{39}$$

$$\mathrm{d}\tilde{\boldsymbol{x}}' = -\tilde{\gamma}\tilde{\boldsymbol{v}}\,\mathrm{d}\tilde{t} + \mathrm{d}\tilde{\boldsymbol{x}} + [\tilde{\gamma}(1+\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}}/c) - 1]\frac{\tilde{\boldsymbol{v}}\cdot\mathrm{d}\boldsymbol{x}}{\tilde{\boldsymbol{v}}^2}\tilde{\boldsymbol{v}} - \tilde{\gamma}\tilde{\boldsymbol{v}}(\boldsymbol{\kappa}\cdot\mathrm{d}\tilde{\boldsymbol{x}})/c\,, \tag{40}$$

where $\tilde{\gamma} = \gamma(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v}/c), \tilde{\boldsymbol{v}} = \boldsymbol{v}/(1 - \boldsymbol{\kappa}\cdot\boldsymbol{v}/c)$. We can read off from the coefficients the synchrony-adjusted forms of the length contraction, time dilation and velocity reciprocity relations.

2.3.2. Group structure and generalised synchrony

Giannoni [65] and Ungar [215] discussed the algebraic properties of this generalised Lorentz transformation. Giannoni gave a group of transformations which allows differing synchrony conventions in any two frames of reference, but requires no restriction on the magnitude of synchrony vector (so $|\boldsymbol{\kappa}| < \infty$) and thus admits infinite speeds, negative speeds (as opposed to velocities), and some conceptually difficult effects such as negative length contractions. Ungar criticised Giannoni's group because it "rules out a causality condition that causes precede effects", and presented a transformation group which has $|\boldsymbol{\kappa}| < 1$ and thus obeys the causality condition. However, Ungar's algebraic structure imposes the same synchrony choice in each reference frame, against the spirit of the conventionalist thesis. And in defence of Giannoni, it should be pointed out that one should distinguish "spatially coincident causality" from "distant causality"; there is no contradiction if an occurrence at P at time t causes another occurrence at $Q \neq P$ at time $t' < t$ because the two different times are measured at spatially different locations. Indeed, as mentioned in Section 1.5.1, such apparent inconsistencies are familiar consequences of the International Date Line for airline travellers.

The rejection of a temporal ordering in distant causality espouses a point of view in which time at any spatial point flows independently of time at other points, with there being no canonical prescription for the way one links the times at spatially separated points. This viewpoint is in accord with the "fibre-bundle" representation of the conventionality of simultaneity suggested by Anderson and Stedman [5]. In this representation, the conventionality of simultaneity is identified with the freedom to choose $\boldsymbol{\kappa}$ as a particular connection in a fibre bundle consisting of a base space of three-space and fibres of the world-lines of particles along which time is represented. Thus a choice of synchronization is a choice of how the different fibres are to be compared.

Since the spinor formalism gives a powerful and elegant way of expressing and combining (standard) Lorentz boosts [143], it is natural to ask (in light of the above generalizations) whether the spinor formalism can be used to express the Lorentz transformations in arbitrary synchrony. This is addressed in Section 2.4.

2.3.3. Electromagnetism in a more general synchronization

The laws of electromagnetism, as of mechanics, remain covariant under a synchrony transformation. Since $\partial_\mu J^\mu = 0, \tilde{\partial}_\mu \tilde{J}^\mu = 0$; since $F^{\nu\mu}{}_{,\nu} = \mu_0 J^\mu$, the corresponding equation holds for the tilded quantities.

For example, the 4-current density has its contravariant timelike component (charge density) affected by synchrony. This leads to another curious paradox: what in Einstein synchronization is current flow without net charge density (perhaps because of countermoving holes and electrons) can in nonstandard synchrony turn into a net charge density. This may be explained by noting that the remote timing operations which must be used to gate the (moving) charge in a given length are affected, and in an anisotropic manner.

As another example, the electromagnetic four-vector potential formed from the electromagnetic scalar and three-vector potentials ($A^\mu = (V/c, \boldsymbol{A})^{\mathrm{T}}$) transforms under synchrony change as does any other 4-vector. Hence the contravariant voltage V, being the timelike component, is synchrony-dependent; the choice of the one-way speed of light affects the zero of voltage by an amount proportional to the component of the vector potential in the direction $\hat{\kappa}$. This indicates a curious parallelism of the two previously very different conventions discussed in the Introduction. (The contravariant 4-potential is arguably more appropriate than the covariant counterpart, since it is linked to the contravariant electric field, then the contravariant Lorentz force and so to the contravariant 4-position, the choice we initially made for interpreting the physical significance of the transformation we discuss. The covariant counterpart has a very different physical interpretation; see Section 1.5.4.) More generally, the gauge invariance of the electromagnetic fields under the gauge transformation $A^\mu \to A^\mu + \partial^\mu \Lambda$ holds in the revised coordinates provided the timelike part of the gauge term $\partial^\mu \Lambda$ is modified appropriately.

The generalised electromagnetic field tensor $\tilde{F}^{\mu\nu} = \tilde{\partial}^\mu \tilde{A}^\nu - \tilde{\partial}^\nu \tilde{A}^\mu$ [4,65] has as its components in the usual representation the synchrony-adapted 3-fields $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{B}}$ (e.g., $\tilde{E}_1 = c\tilde{F}^{01}$). Similarly $\tilde{F}_{\mu\nu}$ contains

$\acute{\boldsymbol{E}}, \acute{\boldsymbol{B}}$ (for example $\acute{B}_1 = \tilde{F}_{23}$). When these are expanded in terms of the old fields they are found to have the form:

$$\tilde{\boldsymbol{E}} = \boldsymbol{E} + c\boldsymbol{\kappa} \times \boldsymbol{B}\,, \qquad \tilde{\boldsymbol{B}} = \boldsymbol{B}\,, \tag{41}$$

$$\acute{\boldsymbol{E}} = \boldsymbol{E}\,, \qquad \acute{\boldsymbol{B}} = \boldsymbol{B} - \boldsymbol{\kappa} \times \boldsymbol{E}/c\,. \tag{42}$$

It is a worthwhile exercise to verify these equations, and to check that the quantities $\boldsymbol{E}^2 - c^2\boldsymbol{B}^2, \boldsymbol{E}\cdot\boldsymbol{B}$ which are invariant under proper Lorentz transformations have corresponding invariants under synchrony transformations, although the choice of contravariant and covariant component must be handled with care. In detail $\tilde{\boldsymbol{E}}\cdot\tilde{\boldsymbol{B}} = \boldsymbol{E}\cdot\boldsymbol{B}$ since the corresponding invariant, $F^{\mu\nu}F^{\alpha\beta}\varepsilon_{\alpha\beta\mu\nu}$, uses only the contravariant components; whereas $\tilde{\boldsymbol{E}}^2 - c^2\tilde{\boldsymbol{B}}^2 \neq \boldsymbol{E}^2 - c^2\boldsymbol{B}^2 = \tilde{\boldsymbol{E}}\cdot\acute{\boldsymbol{E}} - c^2\tilde{\boldsymbol{B}}\cdot\acute{\boldsymbol{B}}$, since the invariant has the mixed form $F^{\mu\nu}F_{\mu\nu}$.

Naturally, one may follow this through to determining the form of Maxwell's equations in vacuo with sources under general synchrony: $\tilde{\partial}_\nu\tilde{F}^{\mu\nu} = \mu_0\tilde{J}^\mu$ and $\tilde{\partial}_{\{\mu}\tilde{F}_{\nu\lambda\}} = 0$ (where { } denotes cyclic permutation of bracketed indices). These therefore take related forms when all quantities are transformed, provided the appropriately contravariant or covariant 3-fields are used:

$$\acute{\nabla}\cdot\tilde{\boldsymbol{E}} = \tilde{\rho}/\varepsilon_0\,, \qquad \acute{\nabla}\times\tilde{\boldsymbol{B}} = \mu_0\tilde{\boldsymbol{J}} + \frac{1}{c}\tilde{\partial}_0\tilde{\boldsymbol{E}}\,,$$

$$\tilde{\nabla}\cdot\tilde{\boldsymbol{B}} = 0\,, \qquad \tilde{\nabla}\times\tilde{\boldsymbol{E}} = c\tilde{\partial}^0\tilde{\boldsymbol{B}}\,.$$

These equations may be readily verified from Eqs. (41) and (35) and give a wave equation in free space whose solutions are indeed waves propagating at the anisotropic speed which we originally imposed [65]. This may help to remove doubt as to whether the detailed dynamics of light propagation imposes a preference on any synchronization scheme. A student who has mastered the consequences of an anisotropic synchronization to this point has passed a significant threshold in understanding which will fortify against the recent erroneous trends in the literature.

2.3.4. Variable synchrony field and ring interferometry

Suppose, finally, that we choose a synchrony vector $\tilde{\kappa}$ which is arbitrary, no longer a constant vector field [4,96]. Under a general transformation of the form

$$\tilde{t} = t - \tilde{\boldsymbol{\kappa}}\cdot\boldsymbol{x}\,, \qquad \tilde{\boldsymbol{x}} = \boldsymbol{x} \tag{43}$$

the incremental transformations are $\mathrm{d}\tilde{t} = \mathrm{d}t - \boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{x}$, $\mathrm{d}\tilde{\boldsymbol{x}} = \mathrm{d}\boldsymbol{x}$ and the metric becomes of the form of Eq. (27) where

$$\boldsymbol{\kappa} \equiv \nabla(\tilde{\boldsymbol{\kappa}}\cdot\boldsymbol{x}) = \tilde{\boldsymbol{\kappa}} + x^i\nabla\tilde{\boldsymbol{\kappa}}_i\,. \tag{44}$$

For a round-trip speed of light to be synchrony-independent, it is necessary for the round-trip time interval

$$\oint_C \frac{\mathrm{d}x}{c(\hat{\boldsymbol{n}})} = \oint_C \frac{(\mathrm{d}x - \boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{x})}{c} \tag{45}$$

$$= \oint_C \frac{\mathrm{d}x}{c} - \frac{1}{c}\int_{A\subset C} \nabla\times\boldsymbol{\kappa}\cdot\mathrm{d}\boldsymbol{S} \tag{46}$$

to be independent of $\boldsymbol{\kappa}$. Because $\boldsymbol{\kappa}$ is irrotational (the gradient of a scalar has no curl) this condition is guaranteed. It is a basic requirement that the results of optical ring interferometry be manifestly independent of such synchronization choices (Section 1.1). Because this synchrony scheme is defined by a unique hypersurface (Eq. (43)), it forms an equivalence relation (Section 2.1.4) and satisfies the round-trip axiom of Einstein and Reichenbach (Section 1.1; see also the discussion of Weyl and Robertson in Section 3.1.2) automatically.

Conversely, any scheme satisfying (as it must) the Einstein–Reichenbach round-trip axiom must correspond to a space–time coefficient $\boldsymbol{\kappa}$ in the metric which is irrotational, and so has the form $\nabla\phi$ where the integral field ϕ may be written $\tilde{\boldsymbol{\kappa}}\cdot\boldsymbol{x}$ (by definition of a suitable, if not a unique, choice of vector field $\tilde{\boldsymbol{\kappa}}$) and thence a synchrony hypersurface (Eq. (43)). Hence conformity to the round-trip axiom will guarantee that any synchrony will be an equivalence relation. The scheme of Section 1.5.1 as a special case certainly satisfies these requirements.

2.4. Spinors

Although the symmetry group of Minkowski space is SO(3,1), a Lorentz transformation can be represented in a 2D complex space, with symmetry SL(2,C), in terms of the matrix representation:

$$\begin{pmatrix} ct+z & x+\mathrm{i}y \\ x-\mathrm{i}y & ct-z \end{pmatrix} \to A\begin{pmatrix} ct+z & x+\mathrm{i}y \\ x-\mathrm{i}y & ct-z \end{pmatrix}A^{\dagger}\,, \tag{47}$$

where $A^{\dagger}$ is the hermitian conjugate of A.

For example, in their development of the spinor formalism for the expression of the Lorentz transformation (in arbitrary synchrony), Penrose et al. [162] began with the future and past null cones at the origin of Minkowskian space–time ($c^2t^2-x^2-y^2-z^2=0$), and then took constant time slices ($ct=\pm1$) to obtain two unit spheres, $S^{\pm}$. They then proceeded to explain how an observer can project what he has seen onto the "past", S^{+}, sphere – a "sky mapping – and how the "future" S^{-} sphere provides a representation of his field of vision – the "anti-sky mapping". The correspondence between the two spheres is given by the antipodal map ($\boldsymbol{X}\leftrightarrow-\boldsymbol{X}$). Either sphere can be identified with the Riemann sphere on an Argand plane, and thus provides a representation of the complex numbers. Penrose et al. [162] showed that the properties of the Argand plane and the Riemann sphere reflect many of the geometrical properties of Minkowski vector space, and that a restricted Lorentz transformation of Minkowski space is uniquely determinable by its effect on the Riemann sphere (and thus null directions). The complex co-ordinate, ζ say, on the Riemann sphere, can be expressed as the ratio of a pair of complex numbers (ξ,η): $\zeta=\xi/\eta$. A transformation of these last two co-ordinates on the Riemann sphere is expressible as the action of a two-dimensional "spin-matrix", A, on a "spin-vector" made up of the complex co-ordinates $(\tilde{\xi},\,\tilde{\eta})^{\mathrm{T}}=A(\xi,\,\eta)^{\mathrm{T}}$. The co-ordinates in Minkowski space–time are expressible in terms of ξ and η and their complex conjugates through:

$$\mathcal{X}=\frac{1}{\sqrt{2}}\begin{pmatrix} ct+z & x+\mathrm{i}y \\ x-\mathrm{i}y & ct-z \end{pmatrix}=\begin{pmatrix}\xi\\ \eta\end{pmatrix}\begin{pmatrix}\bar{\xi} & \bar{\eta}\end{pmatrix} \tag{48}$$

and the above 2×2 unitary representation of a Lorentz transformation follows.

In this, the space–time coordinate $\mathcal{X}=\sigma_{\mu}X^{\mu}/\sqrt{2}$ where $\sigma_{\mu}\equiv(\mathbf{1},\sigma_i)$, and σ_i are the Pauli spin matrices with the algebra $\sigma^i\sigma^j+\sigma^j\sigma^i=2\delta^{ij}$. The transformations of the spin vectors form the two-dimensional complex group SL(2,C), which form a group of conformal transformations, and thus will map a sphere to a sphere.

However, if we now make an arbitrary synchrony choice in each frame, a constant time slice of the null cone no longer gives a sphere (see Eq. (27)). Although the resulting analogues to $S^{\pm}$ can still be given stereographical projections onto the Riemann plane, the transformations between these analogues (which are induced by generalised Lorentz transformations) are no longer conformal, and so cannot be represented by the spin-transformations. It follows that a spinor formalism cannot be used to represent the synchrony-generalised Lorentz transformations.

In the philosophy literature, Zangari [236] claimed that the above result disproved the conventionality of distant simultaneity. His reasoning was that, in relativistic quantum mechanics, spin-half particles are described using the Dirac equation which is necessarily written in terms of spinors and spin matrices. Because the SL(2,C) spinor formalism of the Lorentz transformation cannot handle arbitrary synchrony, he concluded that the Dirac equation is not compatible with arbitrary synchrony. The existence of spin-half particles therefore to Zangari discredits the conventionality of simultaneity.

This merely reveals the need for a revision of the spinor formalism, as for the material of Section 1.5.1, in handling arbitrary synchrony. Gunn and Vetharaniam [76] refuted Zangari's thesis and generalised the Dirac equation to arbitrary synchrony as a straightforward adaptation of the standard theory [19]. The Dirac gamma matrices may be generalised in the obvious way, using $\tilde{\gamma}^{\mu} = (\tilde{\gamma}^0, \tilde{\gamma}) = S^{\mu}{}_{\nu}\gamma^{\nu}$:

$$\tilde{\gamma}^0 = \begin{pmatrix} 1 & -\boldsymbol{\kappa}\cdot\sigma \\ \boldsymbol{\kappa}\cdot\sigma & -1 \end{pmatrix}, \qquad \tilde{\gamma} = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \tag{49}$$

$$\tilde{\gamma}^{\mu}\tilde{\gamma}^{\nu} + \tilde{\gamma}^{\nu}\tilde{\gamma}^{\mu} = 2\tilde{g}^{\mu\nu}. \tag{50}$$

so that the Dirac equation,

$$(i\hbar\tilde{\gamma}^{\mu}\tilde{\partial}_{\mu} - m)\psi = 0, \tag{51}$$

is covariant under synchrony change. It also follows from the operator invariance in Eq. (51) that the Dirac spinor ψ is unaffected by synchrony changes. This is because the components of ψ are not related to four-dimensional space–time but to an independent internal spinor space. Intrinsic spin is an internal property of a particle, and has no bearing on such external issues as the conventionality of simultaneity.

Karakostas [93] counters the Gunn and Vetharaniam [76] refutation of Zangari. Karakostas, while offering his own corrections to Zangari, takes an equally strong anticonventionalist line, and regards Einstein synchronisation as being uniquely vindicated by the inclusion of spinor structure; he believes that "the standard simultaneity relation is... directly imposed by independent physical phenomena". This viewpoint is not borne out by all the elementary considerations analysed above, nor by the following. The essence of spinorial behaviour displayed in say neutron interferometric experiments cannot be affected by a recoordinatisation of time. A Lorentz boost of the observer is sufficient to convert Einstein synchronisation into non-standard synchrony [90], but equally cannot itself imperil the fundamentals of spinor behaviour. The essential arguments of Gunn and Vetharaniam are not affected.

Karakostas assumes such synchrony-dependent concepts as orthogonal tetrads, and (like Zangari) particular choices of gamma matrices which anticommute to form the (synchrony-affected)

Minkowski metric, as if their standard formulation is essential to physics. However the spinor formalism can be integrated into general relativity without restriction on the orthogonality of tetrads. If his claim were true, each of any other internal symmetry (isospin, SU(3), etc.) could itself be regarded as solving the problem of determining the physically preferred choice of synchronisation in an inertial frame, by virtue of the fact that only then is its standard formal description preserved. However, it is an axiom of internal symmetries such as the spinor group that they involve fundamentally different physics, distinct from space–time symmetries. The complexities in the formal description which result from a synchrony change (such as a redefinition of the gamma matrices so as to form the synchrony-dependent metric upon appropriate commutation, as in Eq. (50)) do not affect the essential physics.

2.5. Other aspects of conventionality in measurement

2.5.1. General covariance

Several comments on links with gauge theories were given in Section 1.4.1. In addition, we mention that the relationship between gauge transformations and relativistic test-theories has been discussed intensively. The gauge group of general relativity, the set of diffeomorphisms on the manifold, is commonly taken as relating to active mappings from the manifold to itself (from one point to another), and a coordinate transformation as passive and local, and not a diffeomorphism, although the active and passive transformations can be identified at a coordinate level.

Kretschmann's [103] discussion of general covariance as a mathematical property of any physical theory originated the statement that general covariance as such has no physical content. This popular if not universal view (see Section 1.2) denies a major role for the search for general covariance in Einstein's development of the general theory of relativity. Curiously, this has spawned two almost diametrically opposed positions on the conventionality of simultaneity in the literature.

On one hand, this view has been used as ammunition within an extreme conventionalist position to argue that the conventionality of simultaneity is mirrored in the time transformations that are a subset of general co-ordinate transformations. On this view it is because there is no physical content in clock synchrony change that one has freedom of choice of synchronization scheme [80].

On the other hand, an extreme anti-conventionalist view is to argue that the conventionality of distant simultaneity is a trivial result of the freedom in this co-ordinatization which has no effect on any physical consideration, and in particular has nothing to say about a basic principle (hypothesised in this approach for other reasons) that there is a fully determinable temporal structure in space–time. Following Malament, this structure is taken to be reflected in a certain method of specifying Einstein synchrony from considerations of causality alone (see Section 2.2.1).

Neither of the above extreme positions is compelling. The first fails to do justice to the situation in the context of a theory (such as Newtonian theory) which allows infinite-speed round-trip propagation, and thus a means of determining simultaneity in spite of general covariance. While one is free to choose an arbitrary temporal coordinatization, there is a clear preference for the choice of a coordinatization in which all clocks which are instantaneously comparable show the same time. Reasons for the clear custom of ranking the compulsions of conventions on gauge and on the relativity of simultaneity are discussed in Section 1.4.1. The second position is unwarranted because the implied hypothesis breaches the limitations of what can be measured. Any determination of space–time structure must be empirically based; any conclusion drawn from measurements is necessarily invariant of synchrony, and thus cannot single out any synchrony choice. This is demonstrated most clearly if the analysis is performed in the context of a mathematical formalism which is synchrony-covariant.

The philosophy espoused in this work is that the limitation placed on measurements by the conventionality of distant simultaneity is not merely that which is rooted in co-ordinate freedoms; rather, it is the naturally occurring limitation in comparing distant clocks arising from light being a first signal, and the path dependence of time dilation for moving clocks that provides for under-determination of certain quantities. As Winnie [233] showed, convention and measurement interact.

Zalaletdinov et al. [235] propose a subtle revision of the principle of general covariance. They suggest that from the geometrical point of view one must be able to distinguish between what is in a gauge as a result of a restriction of the diffeomorphism group on a manifold ('active'), and as a result of a general reparametrisation of coordinates ('passive'). Since coordinates are then viewed actively and passively, they cannot be distinguished from dynamical variables. Zalaletdinov et al. [235] aim to remove irrelevant 'degrees of freedom' (such as ficticious forces) for a given problem. They use restricted coordinates (coordinates preferred by the gauge), guided by a principle of restricted (rather than general) covariance which imposes a control on gauge choice, so that the coordinates corresponding to the restricted diffeomorphisms are exactly those used when the diffeomorphism group is so restricted, leaving only the relevant dynamical variables. The challenge is to formulate a selection principle which would guide the choice of an appropriate gauge, for a given set of physical conditions in any situation. The approach of Zalaletdinov et al. [235] suggests the most promising argument for an anticonventionalist position on synchrony of which we are aware, although details would need clarification in that context.

2.5.2. Length conventionality

It is not always obvious whether some quantity is synchrony-dependent or not, and the status of spatial measures in this context has been subject to much debate. Given the importance of spatial measurement in physical theory, questions of synchrony have implications in a diverse number of topics which are seemingly unrelated to simultaneity issues. An example of this is the Ehrenfest paradox, considered in Section 2.6. Another example is the relationship between the concepts of proper length and rest-length. Basri [12] gives a careful account of some basic issues in general relativity.

An arbitrary coordinate transformation (e.g. [81]) will certainly introduce length conventionality as quickly as time conventionality. The covariant form of the simplest synchrony transformation [4] will make the two-way or round-trip speed of light anisotropic (see Section 1.5.4).

Anderson and Stedman [5] discussed a challenge to the conventionalist position arising through the assignment of an operational significance to proper lengths. In a differential-geometric spirit, one can foliate space–time using surfaces of simultaneity. One may choose to identify distances in physical space with the four-dimensional space–time intervals ds, or proper lengths, along surfaces of simultaneity. The result is clearly dependent on the choice of surfaces of simultaneity, and thus on the choice of synchrony. Such a prescription makes a numerical distinction between the "proper length" of a rod and its rest-length.

Contrasting positions have been taken on such an ascription. For example, Reichenbach [178], while supporting the conventionality of simultaneity, considered that the proper length of a measuring rod could only be defined within a simultaneity convention, and indeed defined a characteristic length of a measuring rod as the proper length associated with Einstein synchronization. Nerlich has written at length on synchrony issues, denying that special relativity has a natural foothold for such concepts [153, 154]. Nerlich [152] also considered the proper length of a rod as conventional and made a similar identification of a rod's rest-length and with the proper length arising from Einstein synchronization, but, in a shift away from Reichenbach's position, regarded such an identification as preferring Einstein synchronization. Later, Coleman and Korté [37] claimed that proper length (which they entitled the "spatial metric induced on a hyperplane of simultaneity"), while theoretically dependent on simultaneity convention, is in practice empirically measurable, thus providing an empirical determination of simultaneity relations.

Anderson and Stedman [5, 6] commented that such appeals to proper length as made by Coleman and Korté [37, 38] in order to motivate a unique simultaneity relation are misplaced. Indeed, the Coleman–Korté emphasis on the empirical character of a proper length does not survive the simple observation that any observer using an anisotropy vector $\boldsymbol{\kappa}$ will derive a length equal to another observer's proper length, the latter observer having a relative speed $\boldsymbol{\kappa}c$. There can be no such fundamental distinction between these two kinds of length. Spatial distances can be defined in a synchrony-independent way, in a manner which is in accordance with the standard approach of general relativity, such as given by Møller [148]. That spatial distances can be defined independently of simultaneity choice had already been noted by Havas [81] who not only used Møller's results, but also imposed a Euclidean spatial metric as a necessary requirement.

If one were to assume that, as in the Einstein synchronization case, the two sets of metric components coincide, one would introduce an apparent synchrony dependence into spatial measures. However, Møller [148] had shown that, in the context of general relativistic space–time, the spatial metric tensor components γ_{ij} are given in terms of the space–time metric tensor components $g_{\alpha\beta}$ by

$$\gamma_{ij} = g_{ij} + \gamma_i \gamma_j\,, \qquad \gamma_i \equiv \frac{g_{i0}}{\sqrt{-g_{00}}}\,. \tag{52}$$

In Einstein synchronization, $\gamma_{ij} = \eta_{ij} = \delta_{ij}$. However, in general, the metric tensor components are given by Eq. (27). This still preserves the result [81, 5]:

$$\gamma_{ij} = \delta_{ij}\,, \qquad \mathrm{d}s^2 = \mathrm{d}\boldsymbol{x}^2\,. \tag{53}$$

Thus, the traditional and intuitive Euclidean metric holds for any spatial distances in any synchrony choice, and accords with the operational approach where, under a synchrony change, the spatial separation of two clocks is specified before the clocks are synchronised.

The choice of a synchrony-dependent spatial measure leads to the situation that the numerical value of the distance between two synchronised clocks would change if they were reset using a different synchrony scheme. Such a choice would be a complication if one wished to use devices such as rods for taking spatial measurements.

There is indeed no special status for time here, and some work has been done on the recognition of conventionality in length definitions, particularly in the Russian literature, which documents another such range of controversies [206]. Tyapkin's review [212] is remarkable, not least for the editorial disclaimers which follow it.

2.6. Ehrenfest paradox

Ehrenfest's paradox [47] concerns length contraction effects on a rigid disc which is set in rotation. As seen from an inertial observer at the centre of rotation, the disc's circumference will appear Lorentz contracted because it is moving transversely to the observer. However, because it is not in transverse motion with respect to the observer, the radius of the disc will not appear contracted. Thus the ratio of the disc's circumference to its radius will no longer be 2π, apparently violating Euclidean geometry. Einstein [50] had argued that the paradox was resolvable on the basis that because a rotating disc was non-inertial, its geometry was necessarily non-Euclidean. However, the paradox still remains in the case of an inertial observer viewing the disc, and has been considered by a number of authors offering a variety of solutions, which range from the purely kinematical to the dynamical (e.g., see [25, 27, 72, 185]). Vargas and Torr [218] attempted to discredit the conventionality of simultaneity by claiming that Grøn [72] had resolved the Ehrenfest paradox for the case of special relativity uniquely with Einstein synchronization. Grøn's strategy for resolving the paradox was to show that the motion that would realise contraction of the periphery of the disc is inconsistent with special relativistic kinematics. However, there is a flaw in Grøn's proof, and Vargas and Torr's argument fails. Grøn considered a dust of n particles (representing the periphery of a disc) moving in a circular path, and found that the acceleration programme which would give circular motion to the dust and at the same time keep the rest length between neighbouring particles constant gave kinematically self-contradicting boundary conditions. However, Grøn used the Lorentz transformation from the instantaneous inertial rest frame of an arbitrary pair of neighbouring particles to the inertial rest frame of the centre of the disc using the Lorentz transformation, and this cannot be consistently achieved in a rotating frame [165].

3. Synchrony and experimental tests

3.1. Test-theories of relativity

3.1.1. More early history of test theories

Although the assumptions and postulates used in the theoretical derivation of the Lorentz transformation are based on experimental evidence, there has been great interest in using experiments to test directly the Lorentz transformation. This has encouraged an attempt to measure quantities which are conventional. The two separate considerations of synchrony and the validity of special relativity, which are involved in this area, are often confused.

Formulations of special relativity usually begin with the invocation of the Lorentz transformation to relate any two frames in relative motion. It then follows that the Minkowski metric is an invariant in every frame and is the chosen space–time metric. Therefore, a natural way to test special relativity is to postulate a parameterised deviation from it, most obviously by relaxing the constraint that it is the standard Lorentz transformation which links any two frames of reference. Departing from this standard is to deny the invariance of the Minkowski metric under a boost). This arbitrariness must be constrained in some way by imposing enough structure on the theory to allow useful experimental predictions to be made. Empirical conclusions must be drawn only within the context of these initial assumptions.

Historically, it was the fact that the Lorentz group of transformations was a symmetry group of electrodynamics that led to the acceptance of the validity of the Lorentz transformation for the description of non-gravitational physics in inertial frames. Lorentz showed, in 1904, that the Lorentz group was a symmetry group for Maxwell's equations in vacuum. Although Lorentz did not consider his group as fundamental to nature [3], this role was realised in the following year by Poincaré who showed that electrodynamics was covariant with respect to the Lorentz group [3]. Einstein cemented the fundamental character of the Lorentz group by postulating, first, the principle of relativity: "... that in all coordinate systems in which the mechanical laws are valid, also the same electrodynamical and optical laws are valid ..." [48] and, second, the constancy of the one-way speed of light in vacuum (independent of the motion of the source), in all inertial frames.

The necessity of Einstein's second postulate has been removed as far back as 1911, according to Berzi et al. [18]. The isotropy and homogeneity of space–time show that the transformations have the properties of group elements, and yield the Lorentz and Galilean transformations (and anything in between, i.e. with any value for the limiting speed including the Einstein value of 299 792 458 m/s and the Galilean value of infinity) as the only possible candidates for relating one inertial frame to another [18, 117, 118, 92, 42]. The relativity principle then gives reciprocity. With this approach, the physical difference between these two theories manifests itself in the presence of a parameter identified as the limiting speed for matter. The Galilean transformation results when this limiting speed is taken to be infinite; in general the Lorentz transformation holds. This distinguishes two concepts: the speed of light and the limiting speed of matter [117]. In the TH$\varepsilon\mu$ test theories this distinction is exploited to justify experimental tests of the equivalence of these speeds [64, 231]. Using a different approach again, Lalan [113, 114] derived the Lorentz transformation without using the relativity postulate or reciprocity. Requiring the usual conditions of space–time homogeneity and isotropy of space, he demanded that the transformations formed a parity-invariant group which preserved causality, and arrived at the Galilean and Lorentz groups of transformations [114].

3.1.2. Robertson's test theory

Robertson [183] attempted to infer the Lorentz transformation from experimental observation. His test theory of special relativity had a major influence and in turn motivated the influential Mansouri–Sexl test theory [133–135]. Robertson, with a similar emphasis to Einstein's on the "resting frame" (see Sections 1.1, 1.3.2, 1.5.4), started with a postulated rest-system endowed with a Minkowskian metric, in which light propagated rectilinearly and isotropically. This rest-system was endowed with "preferred" (convenient) physical properties and all analysis was carried out with reference to this frame when physical predictions were made. Assuming isotropy and homogeneity of space–time, Robertson considered a linear transformation (with unknown parameters) linking the rest-system to an arbitrary moving (laboratory) frame. The number of parameters was reduced by making various physical and operational demands. Values were then found for the remaining parameters (as functions of the relative speed of the two frames) by an intricate appeal to the results of the Michelson–Morley, Kennedy–Thorndike, and Ives–Stillwell experiments.

On Robertson's approach, the empirical observations from these experiments are translated into axiomatic statements about kinematical behaviour which then give exact functional values for the parameters in the arbitrary transformation. Experimental uncertainty and the continuous nature of the parameters were not taken into account to give ranges for the parameter values. In effect, observations from a small number of experiments played the same qualitative role in Robertson's test theory as symmetry-related or group-theoretic constraints played in formal derivations of the Lorentz transformation. The more usual approach is to use different experiments to constrain the range of values that test theory parameters can take. Unlike Mansouri and Sexl, Robertson did not have a candidate for the hypothesised rest-frame with a status comparable to the cosmic microwave background [163]. In addition, Robertson was not attempting to put tolerances on the coefficients in the Lorentz transformation.

Robertson's use of a linear transformation ensured that the one-way speed of light is isotropic in all inertial frames. Even in the moving frame S, Robertson explicitly states that the Einstein synchronisation scheme he uses to synchronise each clock in S against the clock at the origin will give Einstein synchronisation in the sense of Section 1.1 for any path in S. However to Robertson, prior to an appeal to the Michelson–Morley experiment, the reciprocal value of light speed so defined may itself be dependent on the direction of the path. Since Weyl's Erfahrungstatsache assumes a similar significance to the Michelson–Morley experiment, it differs in status from the axioms of Einstein and Reichenbach; see the discussion in Section 1.1. In any case all such positions differ from the dual observer of Section 1.5.4 who also has an anisotropic round-trip speed of light, yet is compatible with the Michelson–Morley experiment.

Within Robertson's approach, however, Einstein synchronisation is therefore built into the "observational" derivation of the standard Lorentz transformation (as opposed to the generalised version of Section 2.3.1) which in turn predicts isotropic one-way light propagation in all inertial frames. Without the initial assumptions which invoke Einstein synchrony, the isotropy of the one-way speed cannot be obtained from the isotropy of the two-way speed. This limitation has been noted by, amongst others, Vargas [217], and Maciel and Tiomno (see, e.g., [128, 129]). In revising the Robertson test theory, Vargas [217] claims to obtain the Lorentz transformation using the same three second-order experiments considered by Robertson, but without making any assumptions about convention. However, both Robertson and Vargas suppose that light propagates isotropically in the "rest" frame used as a reference for physical analysis of the "moving" frame.

3.1.3. Mansouri–Sexl test theory

Mansouri and Sexl [133–135] developed a test theory which refined and extended Robertson's test theory, allowing synchrony to be varied in the laboratory frame. They claimed to analyse first-order experiments as well as those of second order, and discussed the manner in which a variety of experiments constrained the parameters in the theory, and also motivated the experimental comparison of slow clock transport and Einstein synchronization as a test of special relativity. Their test theory has been a very popular choice as a foil for special relativity, for example in the analysis of experimental tests as those of Riis et al. [180] and of Krisher et al. [110].

With the cosmic microwave background now an obvious candidate, allowing quantitative evaluation of some parameters of the theory, Mansouri and Sexl postulated an aether (or preferred) frame, Σ to serve as the rest-system in which (significantly) the speed of light was isotropic and equal to c, and that the equality in Σ of measuring devices of differing composition implied their equality in all inertial frames. Assuming homogeneity of space–time, they derived a linear transformation from Σ (with space–time coordinates $c\tau, \xi$) to the laboratory frame S with coordinates $ct, \boldsymbol{x}$ of the form:

$$\mathrm{d}t = a\,\mathrm{d}\tau + \boldsymbol{\varepsilon}\cdot \mathrm{d}\boldsymbol{x}/c\,, \tag{54}$$

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{b}\cdot(\mathrm{d}\boldsymbol{\xi} - \boldsymbol{v}\,\mathrm{d}\tau)\,. \tag{55}$$

Mansouri and Sexl used capital Roman letters for τ, ξ, we use SI units and have made all model parameters dimensionless. $\boldsymbol{v}$ is the velocity of S with respect to Σ. a is therefore a time dilation parameter, the variation in the laboratory clock increment dt for given preferred frame increment $d\tau$ given that the laboratory clock is fixed in that frame ($\mathrm{d}\boldsymbol{x}=0$). $\boldsymbol{b}$ is similarly a length contraction matrix, which is diagonal when the axes are aligned (and with $\boldsymbol{v} \parallel \hat{\boldsymbol{x}}$): $\boldsymbol{b}=\mathrm{diag}(\beta,\delta,\delta)$.

Unlike Robertson, Mansouri and Sexl make no assumption about the speed of light in the second frame; the vector parameter $\boldsymbol{\varepsilon}$ – which otherwise bears no connection with the Reichenbach–Grünbaum parameter ε of Eq. (9) – varies with synchrony choice. Mansouri and Sexl investigated the functional values $\boldsymbol{\varepsilon}$ would take (we call them $\boldsymbol{\varepsilon}_{\mathrm{E}}$ and $\boldsymbol{\varepsilon}_{\mathrm{T}}$, respectively) under Einstein synchronization and slow clock transport. The theoretical framework is sufficiently general to make these two schemes inequivalent synchrony methods:

$$\boldsymbol{\varepsilon}_{\mathrm{E}} = \left(\frac{-av}{\beta c(1-v^2/c^2)},\ 0,\ 0\right), \tag{56}$$

$$\boldsymbol{\varepsilon}_{\mathrm{T}} = \left(\frac{c}{\beta}\frac{\mathrm{d}a}{\mathrm{d}v},\ 0,\ 0\right). \tag{57}$$

Agreement of slow clock transport and Einstein synchronization was then taken to require that $\boldsymbol{\varepsilon}_{\mathrm{E}}=\boldsymbol{\varepsilon}_{\mathrm{T}}$ and so that the time dilation parameter must take on its standard value $a=\sqrt{1-v^2/c^2}$, thus pinning down a parameter in the Lorentz transformation. Experimental evidence for the equivalence of slow clock transport and Einstein synchronization to some precision was then taken to bound any possible deviation of a from this value. For this purpose a was expanded in even powers of v/c: $a=1+\alpha v^2/c^2+\cdots$, the first (second-order) parameter α then eventually being constrained to the expected value -0.5 to within experimental error.

The particular results for the one-way speed of light in S arising from differing synchrony conventions are easily obtained by substituting the corresponding values of $\boldsymbol{\varepsilon}$ into the synchrony-generalised Mansouri–Sexl test theory. The light speed is given by [222]

$$c/c(\boldsymbol{p}) = \boldsymbol{\varepsilon}\cdot\boldsymbol{p} + a(\mathrm{d}\gamma^2 + k), \tag{58}$$

$$\mathrm{d}(\boldsymbol{p}) \equiv b^{-1}\boldsymbol{p}\cdot(\boldsymbol{v}/c + (1-\boldsymbol{\kappa}\cdot\boldsymbol{v}/c)\boldsymbol{\kappa}), \tag{59}$$

$$1/\gamma^2 \equiv (1-\boldsymbol{\kappa}\cdot\boldsymbol{v}/c)^2 - v^2/c^2. \tag{60}$$

In particular, when Einstein synchronization is chosen, $\boldsymbol{\varepsilon}$ takes on the value $-a\,\mathrm{d}\gamma^2$, cancelling out any linear dependence on direction and hence on sense. However the term ak contains squares of direction-dependent terms, as can be seen from Eqs. (58)–(60).

As Mansouri and Sexl [133] showed, with Einstein synchronization the degree of this direction-dependence is governed by the length contraction factors and is independent of a. However, the opposite is true if slow clock transport is used: Mansouri and Sexl give, to first order in velocity, an appropriate equation for the one-way speed of light in S. This takes the form [133, Eq. (6.16)]

$$c(\theta) = c - v(1+2\alpha)\cos\theta, \tag{61}$$

where θ is the angular deviation from the x-axis. Since this apparently shows a potential direction-dependent speed of light, Mansouri and Sexl conclude from here that the "one-way velocity of light is a measurable quantity in this case". However, any measurement in this situation is contingent upon the prior assumption of slow clock transport and so the implied choice of one-way light speed. Although slow clock transport synchronization can differ from Einstein synchronization and so yield anisotropy in the one-way speed of light, this is a formal link only, and not on that account an objectively measurable quantity.

3.1.4. Mansouri and Sexl's interpretations

The main problem in this analysis is that of interpretation: to imagine that one has indeed experimentally, empirically or objectively verified some gauge-dependent formula. The presumed absence of a first-order term in the expansion of the time dilation factor in v/c should be contrasted with the more general form of Eq. (15), where a linear term is essential. The assumption of isotropy in Σ is responsible for the lack of generality of the Mansouri–Sexl formalism and underlies this common deficiency of interpretation. If this is borne in mind, there is no problem with the Mansouri–Sexl theory; the isotropy assumption in Σ is economical in helping to reduce the parameters of the theory. A simple resolution of the matter is to accept the Mansouri–Sexl formalism with this caution.

However in practice the more far-reaching consequences of such assumptions, as illustrated in Section 1.6 for example, have been ignored or denied to the point of major confusion of the literature. In the second paper of their series (on first-order tests) Mansouri and Sexl [133] explicitly rejected Karlov's [94–96] interpretation of the Rømer experiment, which in fact correctly handled conventionality issues. Hence, as stated in Section 1.3.2, the correct interpretation of the Mansouri–Sexl test theory was obscured at the very start of its historic and continuing reign.

This has seriously compromised the validity of the conclusions drawn by the original authors and by all subsequent users of their test theory. For example, Mansouri and Sexl considered experiment to confer a special status on slow clock transport synchrony. It might rather have been argued that because slow clock transport synchrony has the potential to disagree with Einstein synchrony in the Mansouri–Sexl framework, its claim to fundamental status has been lessened rather than strengthened as a result of their work. As it is, their claims have been widely accepted and the conventional content suppressed. The empirical status of the one-way speed of light itself has been "deduced", by Mansouri and Sexl themselves, and by many others (see Section 3.3). Similarly, the common claim here and elsewhere (see the references on past debates in Section 1.3.2) to "first-order" tests, which are closely related to the claim of the measurability of one-way anisotropies, are seriously flawed.

In the remainder of this subsection, we work through in more detail what Mansouri and Sexl might have attempted to accomplish. In Section 3.2 we review a revised theory [222] which makes all synchrony dependence explicit and, in thus illustrating a more nearly gauge-invariant approach, avoids these interpretative errors.

Though not of pivotal significance in this general context, it may be helpful to note some of the inadequacies in the Mansouri–Sexl attempt to rebut Karlov. These points illustrate the kind of changes that are necessary for a correct formulation of the problem. In Karlov's special relativistic version, Jupiter and the Sun are at rest in an inertial system, and an observatory on Earth measures the delay in reception of periodic signals from Jupiter as the Earth orbits the sun. The result $\tau_r - \tau_r' \approx d/c$ is deduced, where the left hand side is the cumulative delay of signals relative to the expected time and d is the diameter of the Earth's orbit. It is then helpful to regard the observer's time measurements as equivalent to those from a network of slow-transport–synchronised clocks, the Earth's velocity being considered only to first order, giving the resolution of Section 1.6. In the version considered by Mansouri and Sexl, the Earth is at rest in S, but moving at a velocity $\boldsymbol{v}$ with respect to the aether frame Σ. Jupiter and the associated clock (its moons) orbit the Earth. Mansouri and Sexl [134] use their result for the one-way speed of light resulting from the use of slow clock transport (Eq. (61)) where θ is the angle between the direction of the light and to the velocity $\boldsymbol{v}$ of S with respect to Σ. On this model, all measurements are made using one clock stationary in S; it would therefore have been more appropriate to use the more general Eq. (6.15) [133] (note that in this equation, as well as in Eq. (6.17) of the same paper, d^2 should be read as d^{-2}), which has in it the unknown synchrony parameter ε (whose functional dependence on the parameters a, b, and d is governed by synchrony choice). Then their analysis of Rømer's experiment would not yield their stated result, unless a synchrony assumption is made. In addition the expansion of a by Mansouri and Sexl is not restricted to even powers of v/c for non-Einstein synchronization in the rest frame.

In the second of the first order tests Mansouri and Sexl consider a rotor experiment in which a rotating source and absorber are equidistant from, and on opposite sides of the point of rotation, with which they are collinear. Mansouri and Sexl modify an equation derived by Møller [147] for the Doppler effect in classical aether theory, adapting it to their theory by replacing the Galilean expression for the speed of light ($c(\theta) = c - \boldsymbol{n} \cdot \boldsymbol{v}$) by the expression derived in [133] for the slow clock transport speed of light, $c(\theta) = c - (1+\alpha)\boldsymbol{n} \cdot \boldsymbol{v}$ where $\boldsymbol{n}$ is the direction of light propagation from source to absorber, and $\boldsymbol{v}$ the velocity with respect to Σ of the centre of rotation. The Mansouri–Sexl formula $\nu/\nu_0 = 1 + 2(1 + 2\alpha)\boldsymbol{u} \cdot \boldsymbol{v}/c^2$ (where $\boldsymbol{u}$ is the instantaneous velocity of the absorber) for the ratio of the detected frequency to the emitted frequency predicts a non-null result for transverse Doppler effects unless the time dilation parameter approaches the special relativity value (the special theory of relativity predicting no frequency shift; see Misner et al. [143, p. 63]). It is appropriate to use the slow clock transport formulae for the rotor situation because the ends of the rotor, at which measurements are being made, are moving slowly with respect to the inertial frame of the rotor centre. The rotor experiment supports the validity of special relativity, but cannot determine either the time dilation factor (because this is dependent on the conventional assumption of Einstein synchronization in the aether frame) or the one-way speed of light in the moving frame, because this is again conventional, the choice of slow clock transport being conferred by the experimental set-up.

In the third paper of the series, Mansouri and Sexl [135] analyse the Kennedy–Thorndike and Michelson–Morley experiments. These second order experiments involve to and fro light trips, and hence synchrony does not enter into the analysis of these experiments. As in their analysis of Rømer's experiment, Mansouri and Sexl impose a synchrony choice in Σ, choosing Einstein synchronization by using Eq. (6.17) of [133]. For them then to regard measurements of the isotropy of the return trip speed of light as giving an indication of the isotropy of the one-way speed of light is simply unjustified.

It is interesting to note that the philosophy espoused by Mansouri and Sexl [133] evolved from the starting position of acknowledging the conventionality of simultaneity to the opposing position that the each theory has associated with it a uniquely determinable synchrony convention. The vector $\boldsymbol{\varepsilon}$ links the synchrony choices in Σ and S, and this conventionality excludes its measurement, although its functional form *within* a synchrony choice can be evaluated. Similarly the parameters a and b are dependent on the conventionality in Σ and so are determinable only within a synchrony convention. Rather than values for test theory parameters defining a unique synchrony, the values for the parameters are determined only after a synchrony is defined. The complications arising with conventional parameters when making approximations in analyses are dealt with in section 3.3.

Mansouri and Sexl [133] and Mansouri [136] acknowledged the conventionality of synchronization in a laboratory frame S through the introduction of their parameter ε. The (logically distinct) conventionality of synchronization in the preferred frame Σ is of equal significance. Mansouri and Sexl [133] simply chose Einstein synchronization in Σ. While such gauge fixing is perfectly acceptable in analysing experiment, it obscures the conventional content of the formalism, in particular that of the claim to test the isotropy of the one-way speed of light. The Mansouri–Sexl ε is not purely dependent on synchrony choice in S, but also on that of Σ, as is shown in Section 3.2, where the Vetharaniam–Stedman [222] generalization of the Mansouri–Sexl test theory is discussed.

3.1.5. Following developments

Maciel and Tiomno [128, 129] consider that many of these test theories are, in fact, special relativity "in different coordinate systems" because they agree kinematically with special relativity. Indeed, Mansouri and Sexl describe under the heading "Relativity without relativity" the transformation [133, Eq. (4.1)],

$$\mathrm{d}t = (1 - v^2/c^2)^{1/2}\,\mathrm{d}T\,, \tag{62}$$

$$\mathrm{d}x = (1 - v^2/c^2)^{-1/2}(\mathrm{d}X - v\,\mathrm{d}T) \tag{63}$$

as corresponding to an aether theory which is kinematically equivalent to special relativity. Events which are simultaneous in one frame are simultaneous in the other: $\mathrm{d}t = 0 \rightleftharpoons \mathrm{d}T = 0$. The kinematic equivalence is seen by noting this transformation can be derived from the standard Lorentz transformation by the synchrony transformation $t \to t + vx/c^2$ where v is the relative speed of the frames in Einstein synchronization. To equate the former of the two transformations with an absolute frame theory, and the latter with special relativity is to misunderstand the role of conventionality.

However, Maciel and Tiomno [128, 129] seem to take exception to the use by Mansouri and Sexl of special relativistic values of time dilation and length contraction factors in such an "aether" theory. This criticism is unjustified; e.g., one Lorentz aether theory discussed by Erlichson [58] is compatible with the Lorentz transformation, although it subscribes to a preferred frame. Spavieri [197] also mistakenly regarded simultaneity conventions as distinguishing theories. Mansouri and Sexl also reach our conclusion of the indistinguishability of the above two theories, although for reasons other than the immeasurability of the one-way speed of light. The function of the Mansouri–Sexl type of test theory is not so much as a test for a preferred frame as a test of Lorentz invariance.

3.1.6. Later development of the Mansouri–Sexl test theory

Both the Robertson [183] and Mansouri–Sexl [133] test theories are restricted to comparing frames in uniform motion with respect to a preferred frame. Although this is a satisfactory arrangement for many situations, certain experiments (such as the two-photon absorption experiment of Section 3.3.1) require a more general framework in order that they be more accurately modelled than the Mansouri–Sexl test theory would allow. This limitation was recognised by Abolghasem et al. [2] who extended the Mansouri–Sexl formalism, deriving a transformation from an inertial, aether frame to a constantly rotating frame. These authors also investigated Einstein synchronization and slow clock transport in rotating frames, arriving at the result that, as is the case in inertial frames, those two synchrony change schemes are equivalent only if special relativity holds true. Such an equivalence does not imply a preferred synchrony scheme for the same reasons that a similar equivalence in inertial frames would fail to establish a unique simultaneity relation: the equivalence of the two holds within any synchrony change scheme, and cannot be used to falsify any particular convention.

While the Abolghasem et al. theory extends the work of Mansouri and Sexl [133] to rotating frames, it also neglects the conventionality of simultaneity in the aether frame, thus again suppressing the conventionality of the test theory parameters. The test theory in Section 5.2.1, developed from differential geometric methods by Vetharaniam and Stedman [224], incorporates arbitrary, space-varying synchrony change in all frames, and also allows the laboratory frame to exhibit arbitrary non-inertial motions. The synchrony extension in this last test theory sheds light on the operational significance of the various parameters in the Mansouri–Sexl test theory; the generality of the motions allowed enables more accurate modelling of experiments.

3.2. Recasting of the Mansouri–Sexl test theory

The conventionality of the one-way speed of light is shown below by recasting (both correcting and generalising) the Mansouri–Sexl test theory for general synchrony choice in Σ as well as S. This is developed to verify that the results of experiments (e.g., [180, 110] which involve a local comparison of synchronization convention [64]) are not affected by gauge fixing.

We generalise Eqs. (54), (55) as

$$\mathrm{d}\tilde{t} = \tilde{a}\,\mathrm{d}\tilde{\tau} + \tilde{\boldsymbol{\varepsilon}} \cdot \mathrm{d}\boldsymbol{x}/c\,, \tag{64}$$

$$\mathrm{d}\boldsymbol{x} = \tilde{\boldsymbol{b}} \cdot (\mathrm{d}\boldsymbol{\xi}/c - \tilde{\boldsymbol{v}}\,\mathrm{d}\tilde{\tau})\,, \tag{65}$$

where much as before Σ is a preferred frame with space–time coordinates $\boldsymbol{\xi}, \tilde{\tau} = \tau - \boldsymbol{\kappa}_0 \cdot \boldsymbol{\xi}$, and the laboratory frame S has coordinates $\boldsymbol{x}, \tilde{t} = t - \boldsymbol{\kappa} \cdot \boldsymbol{x}$. Tildes denote that terms are now explicitly synchrony-dependent through the choice of the synchrony vectors $\boldsymbol{\kappa}_0, \boldsymbol{\kappa}$ in Σ and S, respectively. For example, since $\tilde{\boldsymbol{v}}$ is the velocity of S as measured in Σ, it is of the form of Eq. (10): $\tilde{\boldsymbol{v}} = \boldsymbol{v}/(1 - \boldsymbol{\kappa}_0 \cdot \boldsymbol{v}/c)$. A comparison of Eqs. (54), (55), (64) gives (as in Vetharaniam et al. [225] with some misprints corrected)

$$\tilde{a} = \frac{a}{1 - \boldsymbol{\kappa} \cdot \boldsymbol{v}/c}\,, \tag{66}$$

$$\tilde{\boldsymbol{\varepsilon}} \cdot \boldsymbol{p} = \boldsymbol{\varepsilon} \cdot \boldsymbol{p} + \frac{a\boldsymbol{b}^{-1} \cdot \boldsymbol{p} \cdot \boldsymbol{\kappa}}{1 - \boldsymbol{\kappa} \cdot \boldsymbol{v}/c}\,, \tag{67}$$

$$\tilde{\boldsymbol{b}} \cdot \boldsymbol{p} = \boldsymbol{b} \cdot \boldsymbol{p} - (\boldsymbol{\kappa} \cdot \boldsymbol{p})\boldsymbol{b} \cdot \boldsymbol{v}\,, \tag{68}$$

$$\tilde{\boldsymbol{b}}^{-1} \cdot \boldsymbol{p} = \boldsymbol{b}^{-1} \cdot \boldsymbol{p} + \frac{\boldsymbol{\kappa} \cdot \boldsymbol{b}^{-1} \cdot \boldsymbol{p}}{1 - \boldsymbol{\kappa} \cdot \boldsymbol{v}/c}\boldsymbol{v}\,, \tag{69}$$

for any vector $\boldsymbol{p}$. One may also compare Eq. (64) with Eqs. (39), (40).

Mansouri and Sexl's demonstration that slow clock transport and Einstein synchronization can disagree within their generalised parametrization can itself be generalised [225]. Slow clock transport requires

$$\tilde{\boldsymbol{\varepsilon}}_T \cdot \boldsymbol{p} \approx \tilde{\boldsymbol{b}}^{-1} \cdot \boldsymbol{p} \cdot \tilde{\boldsymbol{v}}\frac{\partial \tilde{a}}{\partial \tilde{v}} - \tilde{\boldsymbol{b}}^{-1} \cdot \boldsymbol{p} \cdot (\boldsymbol{\kappa} - \boldsymbol{\kappa} \cdot \tilde{\boldsymbol{v}}\tilde{\boldsymbol{v}}/\tilde{v}^2)\frac{\partial \tilde{a}}{\partial q}\,, \tag{70}$$

where $q = \boldsymbol{\kappa} \cdot \tilde{\boldsymbol{v}}/\tilde{v}$. Einstein synchronization requires [225],

$$\tilde{\boldsymbol{\varepsilon}}_E \cdot \boldsymbol{p} = -\tilde{a}\tilde{b}^{-1}\boldsymbol{p} \cdot \frac{(1 + \boldsymbol{\kappa} \cdot \tilde{\boldsymbol{v}})\boldsymbol{\kappa} + \tilde{\boldsymbol{v}}}{(1 + \boldsymbol{\kappa} \cdot \tilde{\boldsymbol{v}})^2 - \tilde{v}^2} . \tag{71}$$

Eqs. (70), (71) show that in general, slow clock transport and Einstein synchronization produce different simultaneity conventions. Equating the two corresponding values of ε gives a differential equation whose most general solution [225] is equivalent, as it should be, not to the standard time dilation factor but to the *explicitly conventional and generalised* formula of Eq. (29):

$$\tilde{a}_{\mathrm{TE}} = 1/\tilde{\gamma} = \sqrt{(1 + \boldsymbol{\kappa} \cdot \tilde{\boldsymbol{v}}/c)^2 - \tilde{v}^2/c^2} . \tag{72}$$

Despite the apparent $\boldsymbol{\kappa}$ dependence of Eq. (71), the synchrony convention in S is independent of $\boldsymbol{\kappa}$; $\tilde{a}$, $\tilde{b}$, $\boldsymbol{\kappa}$ and $\tilde{\boldsymbol{v}}$ are measured in Σ.

3.2.1. Power law expansions and synchrony independence

One synchrony-related aspect in experimental analysis which has considerable potential to mislead is the making of approximations, when consistency with regard to synchrony is often difficult to achieve. Unless all approximations are independent of synchrony choice, misleading results may be obtained, and a synchrony-dependent approximation may disguise the synchrony-invariance of measurable quantities. This is the case both with Mansouri and Sexl [133] and with Will [230], who make approximations to first or second order in a speed which is measured using a given synchrony scheme involving an arbitrary $\boldsymbol{\varepsilon}$, referring to the speed as small. The Mansouri–Sexl formulation contains an expansion in powers of a velocity which is synchrony dependent (on the choice of gauge in S). Isolating individual terms in such an expansion immediately introduces a lack of synchrony invariance into analyses, and can be justified only if the gauge is fixed in the *relevant* frame (with the result that conventionality is obscured). For example, using the Mansouri–Sexl expansion, Will [230, Eq. (3.5)] gave an expression of the following form as an approximation of $A'(\boldsymbol{u})$, the time dilation factor for a frame S' moving at velocities $\boldsymbol{u}$ and $\boldsymbol{W}$ in S and Σ, respectively:

$$\begin{aligned} A' &\approx A(v)[1 + (2\tilde{\alpha} - \Gamma^2)(A/\beta)\boldsymbol{V} \cdot \boldsymbol{u} + \mathrm{O}(u^2)] , \\ \tilde{\alpha}v &= \tfrac{1}{2}(A^{-1}\partial A/\partial V + \Gamma^2 V) . \end{aligned} \tag{73}$$

Note that the symbols in this equation have been translated from Will's notation into the notation of this section: Will's $\boldsymbol{w}$ and $\boldsymbol{v}$ in his paper correspond respectively to the $\boldsymbol{V}$ and $\boldsymbol{u}$ conventions used by Mansouri and Sexl and adopted here. The quantity β is the length contraction in the direction of $\boldsymbol{V}$. Will derived his expression by approximating to first order in u, which is dependent on the synchrony $\boldsymbol{\varepsilon}$ in S. It is easily seen that the left hand side of Eq. (73) is independent of ε because it contains just the time dilation factor for S' as measured in Σ. However, the right hand side of that equation contains the velocity $\boldsymbol{u}$, which is intriniscally dependent on $\boldsymbol{\varepsilon}$ because $\boldsymbol{u}$ is measured in S. Now, none of the other quantities in the equation are dependent on $\boldsymbol{\varepsilon}$, and so the right hand side is $\boldsymbol{\varepsilon}$-dependent. Will's equation suggests that an observer in Σ can, by making measurements, determine a "true" synchrony for S. This would be physically impossible, and the disparity in the expression is the result of failing to maintain synchrony-invariance throughout a calculation. It is also the case that a speed which is small in one synchrony scheme may be very large in another, and thus if one is dealing with arbitrary synchrony, it may be inappropriate to make an approximation to first-order in velocity. A synchrony-independent Taylor's series expansion of $a(\boldsymbol{w})$ about $\boldsymbol{v}$ is possible; the terms in the expansion will vary differently from each other under a synchrony transformation in Σ. So an approximation to $a(\boldsymbol{w})$ may not exhibit the same synchrony-covariance as $a(\boldsymbol{w})$. This discrepancy is unavoidable in such an approximation. However it is minor in its effect when compared with the inappropriate introduction or deletion of synchrony-dependence on only one side of an equation: the κ-dependence of a – and hence its conventionality – has been preserved while at the same time no spurious synchrony-dependence has been inserted. A fuller analysis is given in Vetharaniam et al. [225].

3.3. Recent experimental "tests of special relativity"

The test theories produced by Robertson [183] and Mansouri and Sexl [133] have motivated many experimental tests of various special relativistic predictions in the sense that these test theories (or modifications of them) are used as a framework for analysis. Most currently available physical interpretations of the results of such experiments erroneously attribute a measurable status to the conventional quantities in the test theory being used; authors may acknowledge a role to conventionality, and then embrace an interpretation which effectively denies its role. This follows the precedent set explicitly by Mansouri and Sexl [133].

MacArthur et al. [127] analyse (within the Robertson formalism) an interesting experiment in which a beam of hydrogen atoms in their ground state is intersected at a variable angle θ by an ultraviolet laser beam whose ionization effects on the hydrogen atoms are measured. The ratio of the energy of the laser beam as seen by the atoms to its rest frame energy is obtained by the authors to be proportional to $(1+\beta\cos\theta)$ where by varying the angle θ of intersection of the two beams, one can test the sinusoidal variation predicted by the authors. This is not a test of special relativity because this variation is universally predicted. The authors acknowledge the Mansouri and Sexl test theory, but chose to work in Robertson's less comprehensive test theory. Presumably MacArthur et al. [126] follow MacArthur [127] who considers the test theories of Robertson [183] and Mansouri and Sexl [133] equivalent on the grounds that Einstein synchronization in both frames of the Mansouri and Sexl test theory produces a test theory which can be identified with Robertson's.

Robertson [183] assumes isotropy of light in the moving frame; Mansouri and Sexl [133] make no such assumption. Maciel et al. [128, 129] also dismiss the handling by MacArthur et al. [127] of absolute time for the Doppler and lifetime experiments: whereas the test theories being discussed provide an "aether" transformation from a generally non-accessible preferred frame to an arbitrary one (requiring one, when considering two different moving frames, to transform from one to the other via the preferred frame), MacArthur [127] (and also [126]) use the aether transformation to link directly two accessible frames (for example the laboratory rest frame and rest frame of atomic beam).

Mansouri and Sexl's erroneous belief that the one-way speed of light could be measured empirically in applications of their theory has been inherited by a surprising number of good physicists. We mention Vargas [217], also Will, Krisher and coworkers [108, 110, 229, 230]. Riis et al. [180] title their paper "Test of the isotropy of the one-way speed of light..." and Gabriel and Haugan [64] retain similar terminology. Such literature has played a considerable part in publicity within the last decade (see [184, 78, 229, 173, 199, 171]) of modern tests of relativity.

For example, the Mansouri and Sexl formalism was used as the theoretical framework in which experiments are analysed by Hils and Hall [85] who describe an improved Kennedy–Thorndike experiment (using an interferometer with unequal arm lengths to search for sidereal variations between the frequencies of two lasers locked to different references); Einstein synchronization is assumed through the choice of their expression for the one-way speed of light [133, Eq. (6.17)]. The authors state that this experiment allows purely experimental determination of the Lorentz transformation, when in fact the dilation and contraction parameters in the Lorentz transformation are dependent on the synchrony choice in the aether frame, and ε (the moving frame's synchrony vector in the Mansouri and Sexl formalism) is also a conventional quantity.

Kaivola et al. [91] claim to have measured the relativistic Doppler shift for neon. Their experiment actually compares the frequency difference between two lasers, one locked to a two-photon absorption transition in a fast beam of neon, the other to the same transition in thermal neon. A similar experiment is performed by Riis et al. [180] who look for sidereal variation in the frequency difference of a rotating and a stationary laser locked to the resonant frequencies of two-photon absorptions in (different) atomic vapours. It is maintained that the measured frequency variation gives a restriction on the anisotropy of light propagation. The claims made by Riis et al. [180] have already been seriously disputed (see [13, 181]), and for good reason; all experimental measurements are compatible with all synchrony schemes and hence cannot differentiate between different synchrony conventions.

Will [230] stated that a direct measurement of the absolute value of the speed of light in S between two points will depend on the synchronization of the clocks, but that "a test of the *isotropy* of the speed between the same two clocks as the orientation of the propagation path varies relative to Σ should not depend on how they were synchronized. ..." Will also stated that experimental results should not depend on synchronization procedures, so one would understand that the measurables in the test referred to above are of a synchrony-invariant nature.

Like Mansouri [136], Will noted that observables cannot be affected by synchrony choice within the laboratory frame. He accepted that a measurement of the one-way speed of light in the laboratory frame using "a time-of-flight technique" between two clocks is synchrony dependent, but stating that " ... a test of the isotropy of the speed between the same two clocks as the orientation of the propagation path varies relative to Σ should not depend on how they were synchronized, ..." he maintained that this allows a determination of the isotropy of one-way light speeds. However, this argument neglects the effects of a synchrony choice, within the Mansouri–Sexl test theories or even within special relativity, on the cumulative time dilation experienced by a slowly transported clock [222, 233]. The net change in synchrony change induced under slow clock transport is itself synchrony dependent in such a way as not to affect experiment. It is precisely synchrony invariance which prevents an experimental determination of conventional quantities.

Will attempted to distinguish a measurement of the value of the one-way speed of light from a test of the isotropy of the one-way speed of light, claiming that the former is conventional, but that the latter is not, and is measurable. This claim is unsustainable; one cannot hope even to test the isotropy of the speed of light without, in the course of the same experiment, deriving a one-way numerical value at least in principle, which then would contradict the conventionality of synchrony.

Vetharaniam et al. [223] illustrated this by considering a triangle ABC of paths (Fig. 9). If we can assume with Will that the average speed of light on any round trip (for example c_{ABA}) is c, and the speed of light is isotropic at each vertex, it follows that $c_{AB} = c_{AC} = c_{BC} = c_{BA}$ where we have used isotropy at A, C, B, respectively. Hence since the one-way light speeds on any path are equal,

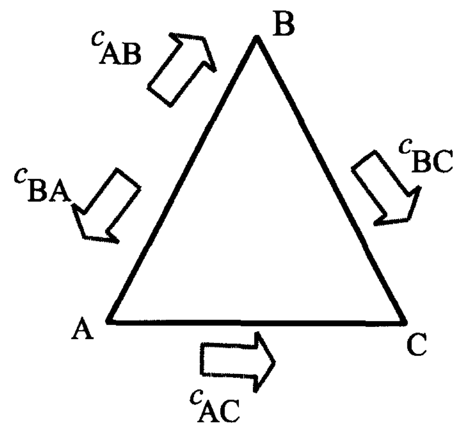


Fig. 9. If the one-way speed of light is isotropic at any point, it is also reciprocal along any segment.

($c_{AB} = c_{BA}$, etc.) each must be equal to c. An independent and objective test of isotropy cannot therefore be possible. All experiments based on a comparison of slow clock transport and Einstein synchronization will, according to special relativity, confirm the apparent isotropy of the one-way speed of light. As an experimental test of special relativity, this is a highly significant result, but as a test of the isotropy of the speed of light it is an illusion.

Several of the experiments mentioned in Section 3.1.1 are analysed by Will [230]: the two-photon absorption, maser phase, Mössbauer-rotor and rocket-redshift experiments. Will pointed out that these experiments have the potential to set bounds on Lorentz-violating, preferred-frame, alternative theories to special relativity. Some incorrect claims for these important experiments as well as some general synchrony considerations are revised in Sections 3.3.1 and 3.3.2 following Vetharaniam et al. [223].

Will [230] also analysed similarly two other experiments – a rocket red-shift experiment [221] and a Mössbauer rotor experiment [28]. These are not re-analysed here, since the examples we discuss should be adequate to clarify the points at stake. However we indicate a synchrony-related point on Will's model for the Mössbauer rotor experiment. In this experiment, an absorber is positioned at the centre of a rotating disc, and measurements are made of the change in transmission of gamma rays through the absorber as a function of the propagation direction of these rays from an emitter placed on the rim of the disc. Will assumed that the disc rotates rigidly in the laboratory frame. This is unjustified, because Will's approach logically demanded the use of an arbitrary synchrony convention, in which a body which rotates rigidly according to some synchrony convention does not preserve the same rigid orientation according to another (see Section 1.6). All assumptions one makes must be synchrony invariant when one is dealing with arbitrary synchrony. The model Will uses for the maser phase experiment is appropriate for the Mössbauer experiment, because that model does not assume rigid rotations of a disc, and its assumptions of relative motions are manifest in the Mössbauer experiment.

Since the analyses of the results of the experiments mentioned above do not take into account synchrony considerations in the hypothesised preferred frame, it is not explicitly obvious that the dilation and contraction factors (the parameters a and b in the Mansouri and Sexl test theory) are dependent on the synchrony choice in the aether frame [222] and thus definitely not measurable, in contrast to the claims of MacArthur et al. [126], Hils and Hall [85], Kaivola et al. [91] and Krisher et al. [110]. As a consequence of this, the Lorentz transformation is not inferable by experiment. And although synchrony choice in the preferred frame does not affect the results of experiments in the moving frame, one should be aware that these results themselves may be dependent on the conventionality in the moving frame in a way which is not immediately transparent. For example, the choice of experimental set-up can induce a synchrony convention which is reflected in the result. This is the case in Rømer's experiment, where in effect slow clock transport is used, as discussed in Section 3.1.1.

Thus the parameters in the frame transformation may be estimated only within the Einstein synchronization "gauge" in the preferred frame; the one-way speed of light is isotropic within the Einstein synchronization gauge in the moving frame (and within the slow clock transport gauge in special relativity). Attempts to experimentally disprove non-Einstein synchronization per se are futile because [227] non-Einstein synchronization simply corresponds to a change of coordinates and, by covariance of physical laws, must be compatible with any experiment compatible with standard synchrony.

3.3.1. The two-photon absorption experiment

The two-photon absorption experiment performed by Riis et al. [180] involved a beam of fast atoms travelling collinearly in a laboratory frame with two counter-propagating laser beams, both produced by one laser. Both beams have the same frequency in the laboratory frame in which the laser was at rest. The frequency of the laser was continually varied (if necessary) to maintain resonance in a two-photon transition between two energy levels of the atoms via an intermediate level, the velocity of the atomic beam being adjusted for resonance in the intermediate state. The variation in laser frequency ν required to maintain resonance in the two-photon transition was recorded. The constraints on the parameters in the transformations, given by a null variation in ν are examined below. In a fully realistic model, the laboratory frame, S, should be accorded variable, non-inertial motion owing to the rotation of the Earth, and the atomic beam should be taken as stationary in S. This was originally simplified so that in the Mansouri–Sexl formalism, S was taken to be in uniform motion with respect to Σ.

Consider an atom with rest frame S' moving at a velocity $\boldsymbol{u}$ with respect to the laboratory frame S. It interacts with two collinear, anti-propagating laser beams which have the same frequency in S, where the laser beams and atomic velocity $\boldsymbol{u}$ are all collinear. Let the atom have three states A, the initial and ground state, and higher states B, C with energies $E_{\mathrm{A}}<E_{\mathrm{B}}<E_{\mathrm{C}}$. The two laser beams (which, in the atom's frame, experience different Doppler shifts) provide the required energies for the transitions A–B and B–C. The laser frequency was continually varied to maintain resonance, and this frequency was examined for a diurnal dependence.

For the atomic transitions to resonate, the frequencies ν'_+ and ν'_- in S' of the two laser beams must be the frequencies associated with the atomic transitions from the initial to the intermediate state and then to the final state: $\nu'_+=(E_{\mathrm{B}}-E_{\mathrm{A}})/\hbar$, $\nu'_-=(E_{\mathrm{C}}-E_{\mathrm{B}})/\hbar$, or vice versa. If $\nu_\pm$ are the corresponding frequencies in S, they are related to the corresponding frequencies in the atomic rest frame by

$$\nu_\pm=\frac{\tilde{a}(\boldsymbol{w})}{\tilde{a}(\boldsymbol{v})}(1-\boldsymbol{\varepsilon}\cdot\boldsymbol{u})\nu'_\pm\,. \tag{74}$$

But, in S, both $\pm$ beams have the same frequency ν. This leads to a relationship between the various frequencies, velocities and factors [225] which is found to be consistent with

$$\tilde{a}(\tilde{\boldsymbol{v}}) = 1/\tilde{\gamma} \equiv \sqrt{(1+\boldsymbol{\kappa}\cdot\tilde{\boldsymbol{v}})^2 - \tilde{v}^2}\,, \tag{75}$$

or Eq. (29) for the special relativistic time dilation in arbitrary synchrony, and with the synchrony-independence of the (locally measured) frequency ν. Thus any experimental test for a diurnal variation of ν cannot measure the time dilation parameter $\tilde{a}$ uniquely, because the latter is synchrony-dependent. Experiment can restrict time dilation only to a class of functions which are related by Eq. (66).

3.3.2. *The maser phase experiment*

In the maser phase experiment of Krisher et al. [110], which Will referred to as the "JPL" experiment, two distant masers (which both output the same rest frequency of 100 MHz) situated at either end of a highly stable 29 km fibre optic cable simultaneously send signals to each other. The geometry of the Goldstone Deep Space Communications Complex is described in Krisher et al. [107–110]. An analyser is situated at each end of the fibre-optic cable. Each analyser is used to compare the phase of the incoming signal with that of the outgoing signal. The observable in this experiment is the relative variation in the phases of the arriving signals. No diurnal variation in this observable was seen.

Krisher et al. [108] (and later Will [230]) claimed that this experimental result constrains the (one-way) time dilation parameter to close agreement with the standard special relativistic value, gives a measurement of the difference between length contraction factors for directions parallel and perpendicular to motion with respect to the preferred frame, and supports the isotropy of the one-way speed of light.

In their treatment, they use a greatly simplified geometry, and in fact assumed one maser to be at rest in the non-rotating frame (S say) comoving with the centre of rotation of the Earth, while the other maser is moving with respect to S (in frame S', say), the latter's motion being due to the Earth's rotation. We make a less crude model by assuming the identity of Earth's centre of rotation with the laboratory frame, S, moving at a constant velocity, $\boldsymbol{v}$, with respect to the aether. The two masers, then, would be tracing a common circular path in S; however their velocities with respect to S can be approximated as being constant over the time periods between the emission and reception of two consecutive signals (wave crests).

Consider an emitter, e, and an absorber, a, at rest in frames S'_e and S'_a, respectively, which are moving at the respective velocities $\boldsymbol{u}_e$ and $\boldsymbol{u}_a$ in the laboratory frame, S. Suppose an observer in S sees two consecutive signals being emitted by e at time t_1 and t_3 from the respective positions $\boldsymbol{x}_1$ and $\boldsymbol{x}_3$. In S these signals will be received by a at two distinct times, say t_2 and t_4, with respective positions of reception, $\boldsymbol{x}_2$ and $\boldsymbol{x}_4$. Let the rest-frequency of the emitter (measured in S'_e) be $\nu'_e \equiv 1/(t'_3 - t'_1)$ and let the frequency of the signals received by the absorber, as measured in S'_a be $\nu'_a \equiv 1/(t'_4 - t'_2)$. These two frequencies are related to the corresponding time measurements in S by

$$\frac{1}{\nu'_e} = \int_{t_1}^{t_3} \frac{\tilde{a}(\boldsymbol{w}_e)}{\tilde{a}(\boldsymbol{v})}(1 - \boldsymbol{\varepsilon}\cdot\boldsymbol{u}_e)\,\mathrm{d}t\,, \qquad \frac{1}{\nu'_a} = \int_{t_1}^{t_3} \frac{\tilde{a}(\boldsymbol{w}_a)}{\tilde{a}(\boldsymbol{v})}(1 - \boldsymbol{\varepsilon}\cdot\boldsymbol{u}_a)\,\mathrm{d}t\,. \tag{76}$$

Now consider the phase comparison made between the signals a receives from e and the signals that a, itself, emits. This comparison is made by a, for whom the phases of the incoming signals are $\phi - 2\pi\nu_a' t_a'$ where ϕ is arbitrary. Since a's own signal has a rest frequency of ν_e', its phase in S_a' is $\theta - 2\pi\nu_e' t_a'$. Thus the variation in phase difference between the incoming and outgoing signals, over a period of $1/\nu_e'$ is

$$\Delta = 2\pi(1 - \nu_a'/\nu_e') . \tag{77}$$

Both ν_e' and ν_a' are invariant of synchrony choice in any frame, and thus the quantity Δ (which is the measurable in the maser phase experiment) is unaffected by choice of κ and ε. Hence experimental measurements of Δ cannot distinguish either a preferred value of κ or a preferred value of ε. It then follows that the maser phase experiment cannot measure the κ-dependent time dilation factor beyond a class of synchrony-dependent functions; nor can it give a measure of the one-way speed of light because this speed is also synchrony-dependent. As before, a fuller study [225] is compatible with a fully synchrony-generalised theory, including Eq. (75), the special relativistic form for arbitrary synchronization. This demonstrates in detail that this experiment has the same limitations as the two-photon absorption experiment, and cannot pick out the functional value corresponding to Einstein synchronization.

There has been some confusion as to the reason for the synchrony-independence of the phase differences of the signals measured at one location. In his analysis of the maser phase experiment, Will [230] said of the phase difference measured at one location:

> "Notice that the result is *independent* of the synchronization procedure embodied in the vector ε. This is because the *initial* relative phase of the two oscillators must be chosen arbitrarily; this is tantamount to choosing a convention for synchronization."

However the initial relative phase of the masers (oscillators) has nothing to do with the synchrony convention in an inertial frame with which they are not co-moving. The reason for the synchrony invariance of the measured phase difference is that only one clock, and not a system of spatially separated clocks is needed for such a measurement. Synchrony considerations do not influence this result.

3.4. What do these experiments test?

Certainly the experiments discussed in this section restrict the Mansouri–Sexl parameters as stated. Since we have shown that the Mansouri–Sexl formalism is unable to sustain the physical interpretations usually placed on the experiments, we may well ask that if these restrictions do not prove what various interpreters claim about one-way quantities, what do they prove?

This question is also suggested by such extensions as Abolghasem et al. [1,2], and is vital to motivate the various experimental tests proposed and performed. One critical question in a generalised theory is what kind of physical principles, when encapsulated in test theories, would allow slow clock transport to disagree with Einstein synchronization. Those principles would then be regarded as tested by the relevant experiments.

It is not easy to get a satisfying answer. As always, any test theory is founded on axioms which remain unquestioned so that other, and arguably less compelling assumptions, may be considered vulnerable to experiment. As Mansouri and Sexl discuss, physical assumptions about the behaviour of standard clocks and rulers under frame change are woven into any such formalism of spacetime measurements from different frames. Some of these assumptions in particular must be taken as axioms if others are to be challenged by the outcome of experiment. We would like to identify those assumptions which can reasonably be regarded as properties of each member of the whole family of theories.

It has been argued that an apparent "preferred frame" effect could conceivably reflect something entirely different, such as a metric change with respect to the local Lorentz frame, to which the lab frame is only an approximation. Mizushima [144, 145] suggested that such experiments as those of the Colorado group (Section 3.3.1) approach the level of detecting the spatial anisotropy of metric changes in g_{11} induced by an orbiting mass (10^{-12} for the sun orbiting the Milky Way and 4×10^{-16} for the Earth orbiting the Sun. Mizushima suggests that had the laser interferometer been mounted with floating mirrors, this would have been seen if the experiment were performed at several locations on Earth and several sidereal times, the preferred direction being that of the velocity of the orbiting body. However, if the observable such experiments measure is the comparison of slow clock transport and Einstein synchronization, and if a metric theory inevitably predicts agreement, this interpretation cannot be viable. One may rather detect the Lense–Thirring field of the Earth's rotation, which is of the order of 10^{-10}. In effect, Mizushima proposes a gravity gradiometer. Tourrenc and Melliti [209] have also commented on Mizushima's interesting but unsubstantiated claims, and the need for a new theoretical framework for the analysis of such experiments. Recently, Tourrenc et al. [210] have developed a test theory to unify Robertson's test-theoretic approach and the PPN formalism, using Einstein synchronisation throughout.

An early argument of Eddington, reproduced in Section 2.1.3, may be taken as a general proof that in a metric theory – one in which space–time has a metric which is locally Lorentzian, the world-lines of test bodies are geodesics of that metric and in local Lorentz frames, non-gravitational laws of physics are those of special relativity – slow clock transport and Einstein synchronization must coincide. A related result is that of Khajehpour and Mansouri [97]. The proof by Abolghasem et al. [1, 2], that this coincidence in an inertial frame leads to a related coincidence in a rotating frame, then reduces to another special case of this result.

Incidentally Abolghasem et al. [1, 2] reinforce corrections such as those by Michel [139], Lichtenberg and Newton [119], by Peres [164], by Grøn [70] and by Ashby and Allan [8] of several related erroneous issues related to synchronization. These include the possibility of first order tests, also a supposed inequivalence of slow clock transport and Einstein synchronization in rotating frames. The latter concern was raised by Chiu et al. [29], Cohen et al. [34, 35] and Rosenblum [186], who however had not properly accounted for the Sagnac effect.

Hence no metric theory is an adequate candidate for a test theory for the experiments discussed in this section. A family of test theories which accommodates the possibility of an inequivalence of slow clock transport and Einstein synchronization has to be a non-metric theory. And indeed the (synchrony-generalised) Mansouri–Sexl theory is such a theory, with no metric structure incorporated, and with a preferred frame. It will not in general have a locally Lorentz metric.

One approach to take is that such a family of theories would be likely to embody a range of physical assumptions which have the potential to challenge the standard form of slow clock transport rather than that of Einstein synchronization. A possible class of theories might then be the various revisions of the hypothesis of locality, according to which an accelerated clock at any instant ticks at the same rate as an instantaneously comoving but unaccelerated clock.

It is fashionable (e.g., [139, 67]) to assume the hypothesis of locality as an inseparable part of special relativity. This is motivated by an assumption or simplification to the effect that the essence of special relativity is that the Lorentz transformation must connect the coordinates of two accelerated (as well as two unaccelerated) frames which have a relative boost (at constant velocity). However, this somewhat obscures the logical distinctions that need to be made with respect to (for example) the status of experimental tests. Møller [148] makes the special status of the hypothesis clear. Alternatives to the hypothesis of locality have been discussed; some early and gross proposals for the revisions of this principle, which would certainly challenge the equivalence of slow clock transport and Einstein synchrony, were discredited by Mainwaring et al. [130]. Mashhoon [137] has presented more sophisticated alternatives to the locality hypothesis, which however require large accelerations to induce substantial effects, and which would give vanishingly small corrections in the limit of slow clock transport. This kind of theory would then need some modification if it is to be a candidate test theory of the required form, vulnerable to the various experiments whose analyses have previously proceeded on the flawed Mansouri–Sexl interpretation.

This interpretative problem is further compounded by the possibility that the hypothesis of locality is implicitly built into the assumptions that all members of the Mansouri–Sexl family make about the effect of frame change on standard clocks. One viewpoint is that all supposedly kinematic theories inevitably make such physical and dynamical assumptions.

Einstein [49] has used another isotropy argument, itself universally ignored as far as we know, to show that any departure from the hypothesis of locality has to be an even function of the acceleration, so that no linear term is possible. This argument has some interest in our context; it has links with that in Section 1 for the simultaneity convention discussed in this script. In the context of a discussion of possible acceleration-dependent length contraction factors Einstein wrote: "... an effect of another kind is impossible for reasons of symmetry ... Acceleration-caused dilations (if such exist at all) must be even functions of γ (the acceleration) ... A specific effect of acceleration on the rate of the clocks ... would have to be of the order of γ^2."

We interpret this argument as follows. Because all directions are operationally symmetric, within the comoving inertial frame there is no physical reason to expect a difference in the instantaneous rate of the accelerated clock with the direction, as opposed to the magnitude, of the acceleration. Any rate change must therefore be an even function of the acceleration, in this and therefore in any other frame.

Einstein's statement can be expected formally to fail in an anisotropic synchronisation scheme. For the identical reason, the same directional symmetry assumption that would forbid a velocity-odd term is formally compromised by a choice of anisotropic synchronisation. Hence the time dilation factor of Eq. (15) acquires a linear as well as quadratic dependence on velocity.

Another viewpoint is suggested by Ehlers et al. [46] who reject clocks as basic tools for setting up the space–time geometry and propose to use light rays and freely falling particles instead (see also [166]). They do this for the reasons that standard clocks alone without light only lead to the Riemannian separation with difficulty; that if the metric is defined by clocks the relation of geodesics to free fall is obscure, and finally that starting from light rays, more particularly from the round-trip speed of light, one can manufacture clocks. The fact that this programme is successful at all, even in standard relativity, suggests that standard clocks might be avoided in the development of test theories of the Mansouri–Sexl type, and that a hypothesis of locality in the axioms of the theory is avoidable.

Another approach is that of Golestanian et al. [69] who look for a geometric structure for space–times that are almost Lorentz invariant within the context of a subset of Mansouri–Sexl test theories and Finslerian geometry, giving a test theory in which constraining the parameters constrains the metric and so the geometry. Here there is very little room allowed for manoeuvre, and the axiomatic and unchallenged aspects of the model considerably outweigh the aspects amenable to experimental constraint. Hence the results of this interesting and brave programme presently seem too contrived to be even tentatively credible alternatives; special relativity deserves a better challenge and clearer triumph than the "also-rans" with nearly Riemannian structure. However, as with quantum theory, whose challengers in the form of quaternionic quantum theory and Weinberg's seriously beleaguered nonlinear theory are also highly contrived, it may be precisely because special relativity is such a ubiquitous and successful theory that it is difficult to formulate a credible alternative.

Another approach is that of Narlikar et al. [149], who consider the possibility that a photon has nonzero rest mass. They suggest that its isotropic speed in a preferred frame such as that of the cosmic microwave background may then translate into an anisotropic speed in another frame such as the Earth's, and that this could be regarded as a natural anisotropic dispersion of the vacuum for light.

This work is particularly interesting for the novel effort it makes to ground any observed effect into more fundamental physics. However, Narlikar et al. take no account of synchronization problems in either frame. Even if we may accept an Einstein synchronization convention in the preferred frame, the synchronization issue is still alive in the new frame, as Mansouri [136] shows in the context of Mansouri–Sexl theory.

Narlikar et al. [149] actually consider a search for an anisotropy in the frequency of light travelling in opposite directions in the laboratory frame (and find encouragement from residuals in the laboratory laser-beating experiments and the apex of the dipole anisotropy in the then-existing measurements of the microwave background, although they misidentified the latter direction somewhat; see Section 5.4). They compare in some detail with the strategy of Riis et al. [180]. We may therefore expect a similar resolution of the synchronization conventionality issue in this case as for Riis et al., so that the inequivalence of slow clock transport and Einstein synchronization could also arise from a non-vanishing photon mass. Incidentally tests of Mansouri–Sexl theories have been proposed which can go well outside the Earth-bound laboratory, to use the data from lunar laser ranging [146]. Lunar laser ranging has now reached a level of accuracy [41] as to permit indirect demonstration of the gravitomagnetic consequences of general relativity, and to demonstrate the spatial isotropy of the gravitational interaction [158].

One guard against being blinkered by the presuppositions of any earlier test theory is the proposal of such an utterly different test theory. Another and a particularly sophisticated example of a test theory is that of Bowes and Jarvis [20], who consider the manner in which a κ-deformed Poincaré algebra is one mechanism of imposing a fundamental length scale. This might affect special relativity and the standard experiments on such length scales.

We conclude that after many years of study, it is still not clear what those "tests of special relativity" which confirm slow clock transport to agree with Einstein synchronization (such as those discussed earlier in this section) yield in terms of more fundamental physical constraints, and what is their discriminating power in more fundamental physics. Several approaches for making contact with more fundamental physics seem possible, but are not yet well studied. At this stage we may well be content with constraining the parameters of a formal family of test theories, ensuring in the spirit of this review that a conservative interpretation is applied where necessary. Golestanian et al. [69] illustrate the point of view that to go further may naturally require introducing some geometrical flavour into the model, without converting it into a full metric theory. Our own work in this direction is detailed in Section 5.2.

4. Synchrony and noninertial observers

4.1. Relativistic noninertial observers

So far, we have concentrated on test theories in arbitrary synchrony for inertial frames in flat space–time. However, a more general test theory is required for many applications. This has stimulated for example the work of Abolghasem et al. [1,2] who consider experimentation on the rotating Earth and other accelerated frames of reference. As another example, our recent precision observation [7,203] of electromagnetic effects such as the Sagnac effect in ring lasers which are derived from noninertial effects such as the Earth rotation raise the question as to what is learned about relativity from searches for a diurnal component of the Sagnac signal. These examples illustrate the value of extending our discussion of test theories and of the inclusion of arbitrary synchrony to an accelerated observer in a curved space–time.

Our development proceeds from a geometrical perspective. Coordinates are still needed to describe events. Investigating the coordinates of an accelerated observer raises two separate problems.

First, the observer's coordinate system is derived from a tetrad (a set of four basis vectors that the observer is postulated to choose and to transport along his world-line in space–time). "Metric coefficients" are defined as the set of inner products of tetrad vectors. This does not imply that this is a metric theory, in the sense of one in which the interval is invariant under a boost. The metric coefficients define, amongst other things, the one-way speed of light at the observer's location. The law of propagation of the tetrad along the observer's world-line must take arbitrary synchrony into account. A modification of the Frenet frame method is used here for this; Kreyszig [104] gives a discussion of the standard techniques.

The second problem is the natural and unrestrictive assignment of the coordinates themselves. In the following sections, the Riemann normal coordinate approach used by Misner et al. [143] for the coordinates of an accelerated observer in general relativity is adapted to produce a prescription that allows one to take arbitrary synchrony into account.

Misner et al. [143] require that their accelerated observer propagates a tetrad, and state the requirements for that tetrad, but do not give a formal analysis of how that tetrad is realised. Scorgie [191] uses the standard Frenet frame formulae to obtain the tetrad for an accelerated observer within special relativity, and then follows Misner et al. [143] in their related analysis, obtaining the observer's metric by appealing to the invariance of the interval in going from an arbitrary inertial frame in which the observer is analysed to the observer's accelerated frame.

Central to both the analyses given by Scorgie and by Misner et al. is the consideration of a (constant-time) three-dimensional slice of space–time as the observer's physical space. This precludes the use of their approaches in considering arbitrary synchrony; slices of constant time are necessarily synchrony-dependent. Similarly, one cannot use the curved space–time prescription given by Misner et al. who, for any point on the observer's world-line, take as constant time curves those geodesics whose tangent vectors at that point have zero temporal component (purely "spatial" tangent vectors).

We first present a desired set of local coordinates which will handle space-varying synchrony (Section 4.2). Section 4.3 contains a generalization of the Frenet frame, and Section 4.4 then combines the previous results to obtain the coordinates of an accelerated observer.

The following convention is used for basis vectors. Both for the case of local coordinates and the accelerated observer, basis vectors are denoted by $\{g_\mu\}$ with the basis vectors at the spatial origin denoted by $\{e_\mu\}$. The standard orthonormal basis vectors are written as $\{\eta_\mu\}$. The corresponding metrics are then written respectively as $g_{\mu\nu}, e_{\mu\nu}, \eta_{\mu\nu}$.

4.2. Local coordinates

The choice of coordinate system has no physical significance and is a matter of convenience for the description of events. However, coordinates do reflect to some extent the assumptions made in this description, and so some freedom in prescriptions for coordinatizing a set of events in a general way can be helpful. In this section, the assignment of coordinates to the neighbourhood of an event is considered by generalising the Riemann normal coordinates method in order to facilitate synchrony considerations. Initially this generalization is discussed on an arbitrarily curved manifold. A manifold with zero curvature is used to obtain an actual set of coordinates in order to produce a coordinate system for an accelerated observer with arbitrary synchrony, for application in Sections 4.4 and 5.2.1.

Riemann normal coordinates form a system of coordinates local to a point on a manifold (assigned in a neighbourhood of that point) and are defined in terms of a vector basis defined at that point. There is no requirement that the manifold be Riemannian or semi-Riemannian for the construction of such a coordinate set. Although this point has no bearing in the present context of the special and general theories – which both assume a Lorentzian metric – it allows the results of this chapter to be generalised for the purposes of producing a test theory of local Lorentz invariance in Section 5.2.1 where a wide range of theories is considered.

The discussion in this chapter will consider only manifolds with torsion-free connections. The existence of torsion does not prevent one from finding a system of normal coordinates centred on a point on a manifold; however, if the connection is torsion-free (and thus symmetric), then there exist normal coordinates such that the connection coefficients vanish at that point [99].

The motivation for Riemann normal coordinates comes from the exponential map (see [99, Section 8]). This is a mapping from the tangent space T_{P} of a point P on a manifold to a neighbourhood of that point, and is defined by

$$\exp \lambda V = \gamma_\lambda \,, \tag{78}$$

where γ_λ is a geodesic starting at P with tangent vector V at P. In particular, the exponential map for the geodesics through P maps the tangent vector to each geodesic at P to the point a unit parameter distance along that geodesic [205].

If a vector basis is defined at P, then normal components along such a geodesic can be defined to be proportional to the components of its tangent vector at P as follows. Consider a point P on a manifold with a torsion-free connection. There exists a neighbourhood, N, of P such that for all Q in N (where Q is distinct from P) there exists a unique geodesic, γ say, connecting Q and P. This follows from the definition of Eq. (78) and the property that the exponential map maps a neighbourhood of the zero vector in the tangent space at a point *onto* a neighbourhood of that point [99, Proposition 8.2].

Let λ be an affine parameter along γ_λ with $\gamma_0 = P$ and $\gamma_\lambda = Q$ and let $\mathrm{d}/\mathrm{d}\lambda_{|\lambda=0} = V = N^\alpha \eta_\alpha$, where the basis vectors η_α are the conventional, orthonormal choice, corresponding to Einstein synchronization at the observer's spatial origin: $\langle \eta_\alpha | \eta_\beta \rangle = \eta_{\alpha\beta}$ and $\eta_{\alpha\beta} = \mathrm{diag}(-1,1,1,1)$ (Section 1.5).

The Riemann normal coordinates X^α centred at P are taken as proportional to the parameter distance λ from P to Q and also proportional to the components of V; that is, $X^\alpha(Q) = \lambda N^\alpha$ (see, e.g., [143, 116]). A different choice of synchrony at P corresponds to a set of coordinates different from the Riemann normal coordinates and the use of an alternative tetrad e_α where

$$\langle e_0 | e_0 \rangle = -1\ , \quad \langle e_0 | e_m \rangle = -\kappa_m\ , \quad \langle e_m | e_n \rangle = \delta_{mn} - \kappa_m \kappa_n\ , \tag{79}$$

for three arbitrary numbers κ_n (playing the same role, as the notation suggests, to the earlier 3-vector of this name in Section 2.3).

The basis vectors, e_α, can be related to a set of orthonormal basis vectors, η_α, by

$$e_0 = \eta_0\ , \quad e_n = \eta_n + \kappa_n \eta_0 \tag{80}$$

with the result $V \equiv N^\alpha \eta_\alpha = V^\alpha e_\alpha$ where

$$V^0 = N^0 - \kappa_n N^n, \qquad V^n = N^n\ . \tag{81}$$

Coordinates $\{x^\alpha\} = \{ct, x^i\}$ can then be assigned to $Q = \gamma(\lambda)$ according to the formula $x^\alpha(Q) = \lambda V^\alpha$. These are related to the 'Einstein synchronization' Riemann normal coordinates, $\{X^\alpha\} = \{cT, X\}$ by $t = T - \kappa_i X^i / c$ and $x^i = X^i$. (On a curved manifold, Einstein synchronization then holds only at P.) The choice of basis vectors in these cases defines the surfaces of simultaneity in the whole neighbourhood N of P and thus determines the synchrony choice for all points covered by the coordinate system. A more general (space-varying) choice of synchrony requires a description of the propagation of the spatial basis vectors (which are tangent to the surfaces of simultaneity at each point on the manifold). Now these propagation laws are defined by and require the knowledge of the connection coefficients at all points, and so cannot be handled by the method mentioned above. Initially, it specifies values for the affinities $\Gamma^\alpha{}_{\beta\gamma}$ at only one point P with geodesic deviation corrections applied later [143].

In flat space–time, one can first define simultaneity relations within this context by specifying how spatial basis vectors propagate along the geodesics, because all curvature effects of the manifold are already known. Now, by definition of the connection coefficients,

$$\nabla_\nu g_\mu \equiv \nabla_{g_\nu} g_\mu \equiv \Gamma^\alpha{}_{\mu\nu} g_\alpha\ . \tag{82}$$

The temporal basis vector g_0 plays no role in determining simultaneity relations, and should be unchanged by synchrony transformations, as suggested by Eq. (80). Thus the choice can be made that g_0 is parallel-propagated along the geodesics emanating from P: $\nabla_\nu g_0 = 0$, which by symmetry of the connection coefficients gives the result that synchrony choice will be independent of time. As is also indicated by Eq. (80), a change in synchrony change alters the spatial basis vectors only in the g_0 direction: $\nabla_n g_m \propto g_0$. These considerations leave unrestricted only the propagation of spatial basis vectors in the temporal direction, and together with Eq. (82) suggest the following general form for the connection coefficients:

$$\Gamma^{\alpha}{}_{\mu 0} = 0\,, \qquad \Gamma^{l}{}_{mn} = 0\,, \qquad \Gamma^{0}{}_{mn} = F_{mn}\,, \tag{83}$$

where the F_{mn} are some differentiable functions of position.

The basis vectors along the geodesics can be obtained from the definitions of the connection coefficients in the following manner. From Eqs. (82), (83), it follows that

$$\frac{\mathrm{D}}{\mathrm{D}\lambda} g_0 = 0\,, \qquad \frac{\mathrm{D}}{\mathrm{D}\lambda} g_n = \frac{\mathrm{d}x^m}{\mathrm{d}\lambda} F_{mn} g_0\,. \tag{84}$$

These differential equations then give

$$g_0 = e_0\,, \qquad g_n = e_n + \int_0^{\lambda} \frac{\mathrm{d}x^m}{\mathrm{d}\lambda} F_{mn} g_0 \,\mathrm{d}\lambda\,. \tag{85}$$

The symmetry of the connection coefficients and the requirement that the last equation be integrable suggest a constraint of the form $F_{mn} = F_{,mn}$ for some differentiable function F.

From here, an appropriate neighbourhood of a point P can be coordinatized by using the above values for the connection coefficients and solving the geodesic equation,

$$\frac{\mathrm{d}^2 x^{\mu}}{\mathrm{d}\lambda^2} = -\Gamma^{\mu}{}_{\alpha\beta} \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\lambda} \frac{\mathrm{d}x^{\beta}}{\mathrm{d}\lambda}\,, \tag{86}$$

to obtain a coordinate set. Hence (details are given in Vetharaniam et al. [224]) one may show that F can be replaced by $f = F(0) - F$, to give the final form of the connection coefficients in Eq. (83) as

$$\Gamma^{\alpha}{}_{\mu 0} = 0\,, \qquad \Gamma^{l}{}_{mn} = 0\,, \qquad \Gamma^{0}{}_{mn} = -f_{,mn} \tag{87}$$

where

$$f(0) = 0\,, \quad f_{,m}(0) = -\kappa_m\,, \qquad f_{,mn}(0) = 0 \tag{88}$$

for some differential function $f = f(x^n)$, and with the coordinates then simplifying to

$$x^0 = \lambda(V^0 - \kappa_n V^n) + f(\{x^n(\lambda)\})\,, \quad x^n = \lambda V^n\,. \tag{89}$$

Similarly, from Eq. (85), the basis vectors along the geodesic become

$$g_0 = e_0\,, \qquad g_n = e_n - (\kappa_n + f_{,n}(\{x^q(\lambda)\}))\, e_0\,, \tag{90}$$

and the metric tensor components (obtained by taking inner products of the basis vectors) are

$$g_{00} = -1\,, \qquad g_{0m} = f_{,m}(\{x^n(\lambda)\})\,, \quad g_{mn} = \delta_{mn} - f_{,m} f_{,n}\,, \tag{91}$$

giving the local one-way speed of light in the direction $\boldsymbol{p}$ as

$$c(\boldsymbol{p}) = \frac{c}{1 + f_{,m} p^m}\,. \tag{92}$$

Although the coordinates of Eq. (89) have been derived in a flat space–time, they provide a valid coordinate choice for an accelerated observer in a space–time of arbitrary curvature. They are used in Section 4.4 as a set of "flat" space–time coordinates for an observer whose accelerations and synchrony are both arbitrary.

4.3. Tetrad propagation

Let P be the observer's world-line. The parameter, t, along P is the observer's proper time. When constructing a tetrad to be propagated with the observer, it is natural to take the temporal basis vector to be the tangent to his world-line. This is because in the observer's rest frame his space–time motion is purely in a temporal direction: $e_0 \equiv \mathrm{d}/\mathrm{d}(ct)$. Given the definition of e_0, the other basis vectors are chosen subject to whatever conditions are required of the coordinate frame.

One way of obtaining spatial basis vectors along this world-line is the Frenet frame method. In this method, a set of orthonormal basis vectors is developed sequentially by requiring that the kth derivative of the world-line lies in the span of the first k basis vectors [98]. (Kreyszig [104] discusses of the usual case where each derivative of the world-line is, by definition, proportional to a basis vector.) An orthonormal basis corresponds to a local imposition of Einstein synchronization; using a modification of the Frenet frame method, one can construct a tetrad which allows more synchrony freedom: instead of the usual orthonormality requirement for the basis vectors of a Frenet frame, the inner product relations of Eq. (79) are used.

From the definition, Eq. (82), of the connection coefficients, propagation of the tetrad along the observer's world-line is given by [143]

$$\dot{e}_\mu \equiv \frac{1}{c}\frac{\mathrm{D}e_\mu}{\mathrm{d}t} = \nabla_{e_0} e_\mu \equiv \Gamma^\alpha{}_{\mu 0} e_\alpha \,. \tag{93}$$

We define in Frenet fashion [98]

$$\dot{e}_\mu = A_\mu{}^\nu e_\nu \,, \tag{94}$$

$$\chi_1 \equiv A_0^1 \,, \quad \chi_2 \equiv A_1^2 \,, \quad \chi_3 \equiv A_2{}^3 \,, \tag{95}$$

where $A_\mu{}^\nu = 0$ if $\nu \geq \min(\mu + 1, 3)$.

Covariant differentiation of the relations in Eq. (79) gives

$$\langle \dot{e}_\mu | e_\nu \rangle + \langle e_\mu | \dot{e}_\nu \rangle = 0 \,. \tag{96}$$

By substituting Eq. (94) into Eq. (96) and solving for $A_\mu{}^\nu$ successively for the cases $\mu = 0$ to $\mu = 3$ one obtains

$$\begin{aligned}
\dot{e}_0 &= \chi_1 e_1 - \chi_1 \kappa_1 e_0 \,, \\
\dot{e}_1 &= [\chi_1(1 - \kappa_1^2) - \chi_2\kappa_2] e_0 + \kappa_1 \chi_1 e_1 + \chi_2 e_2 \,, \\
\dot{e}_2 &= -(\chi_1\kappa_1\kappa_2 - \chi_2\kappa_1 + \chi_3\kappa_3) e_0 + (\chi_1\kappa_2 - \chi_2) e_1 + \chi_3 e_3 \,, \\
\dot{e}_3 &= -(\chi_1\kappa_2\kappa_3 - \chi_3\kappa_2) e_0 + \chi_1\kappa_3 e_1 - \chi_3 e_2 \,.
\end{aligned} \tag{97}$$

(A Frenet frame results when $\kappa = 0$.) Comparing Eq. (97) with Eq. (93) gives values for $\Gamma^\alpha{}_{\mu 0}$ at the origin. These may be expressed as [143]

$$\Gamma_{\mu\nu 0} = \frac{(A \wedge U)_{\mu\nu}}{c^3} + \frac{\varepsilon_{\alpha\beta\mu\nu} U^\alpha \Omega^\beta}{c^2} \,, \tag{98}$$

where U, A and Ω are, respectively, the four-velocity, four-acceleration, and angular velocity four-vectors of the observer, $\wedge$ denotes the wedge product, and the observer's self-measured three-acceleration and spatial angular velocity are $\{c^2\chi_1, 0, 0\}$ and $c\{\chi_3, 0, \chi_2\}$, respectively. No restrictions are placed on the other components of the affine connection by this method. In the case of zero angular velocity the observer is in Fermi–Walker transport [143], and then with the choice of an orthonormal basis ($\kappa = 0$) one obtains the standard description of a Fermi–Walker tetrad.

4.4. *Accelerated observer*

Here the development of a coordinate system for an observer with arbitrary acceleration and rotation is considered within the context of special relativity or general relativity. Metric tensor components are obtained to first order. The coordinate set given in Eq. (89) is used to label events. The main task here is to find a prescription for assigning these coordinates in a consistent manner. The observer's world-line P parameterised by proper time, t, is modelled by a curve on a semi-Riemannian manifold with signature (1,3). It is natural to take P as the time axis for the observer, who is assumed to propagate a tetrad along P according to the modified Frenet frame prescription given in Section 4.3. This tetrad reflects the choice for the one-way speed of light at his spatial origin, which is represented as moving along P on the manifold.

Misner et al. [143], who consider only an observer with an orthonormal frame, assign coordinates in the following manner. They consider all geodesics, at each point P(t) on the world-line, whose tangent vectors at P(t) have no temporal component according to the observer's tetrad at that point (Fig. 10(a)). These geodesics are considered to be curves of constant time t and have spatial coordinates assigned along them which are proportional to the geodesic parameter and the tangent vector components at P(t), in a similar fashion to the Riemann normal coordinates case (see Section 4.2).

This prescription has two properties which preclude its use for formulating coordinates with arbitrary synchrony. First, if an arbitrary tetrad is used, the geodesics picked out as curves of constant time (on the basis of having a tangent vector at P(t) with zero temporal component) will vary with choice of κ. This means that along P an undesirable transformation in spatial coordinates would be associated with any temporal transformation which corresponded to a redefinition of the one-way speed of light. In principle, the spatial coordinates should be independent of a change in clock setting. This problem arises because this prescription is not geometric in nature. A second problem with that prescription is that the tetrad choice determines spatial surfaces (surfaces of simultaneity) and so how clocks are synchronised over these surfaces, including the full length of the geodesics embedded in it, removing any freedom in varying clock settings from point to point.

Since it is convenient to assign coordinates along geodesics emanating from the observer, a geometric property is used to distinguish a set of geodesics along which spatial coordinates are assigned independent of synchrony choice. Once a set of geodesics is chosen, the coordinates assigned along members of this set are those of the corresponding "local coordinates" centred at P(t) (Eq. (89)) with the observer's time t being added to the x^0 coordinate.

There are two sets of geodesics that might be expected to be useful for this purpose: those orthogonal to the observer's world-line (which do not necessarily coincide with those having tangent vectors with no time component) (Fig. 10(b)) and null geodesics.

Of the two sets of geodesics, the null geodesics might initially seem the preferable choice. Their use is natural in that it corresponds to the operational approach of obtaining distant information from

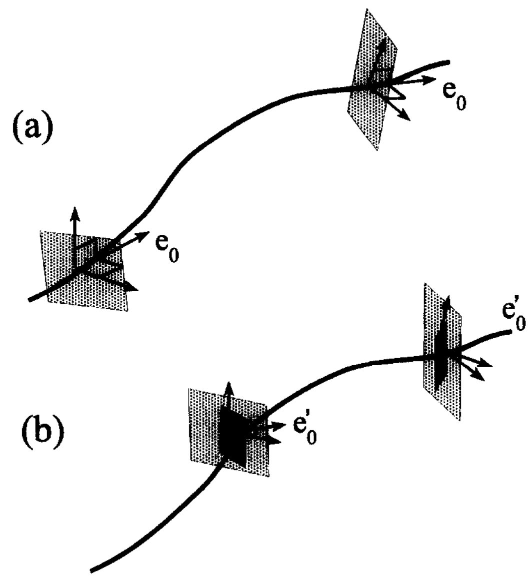


Fig. 10. (a) The Misner et al. construction of geodesics in a constant-time surface. If the time component e_0 of the tetrad at any point is orthogonal to the others, and is tangent to the worldline, this surface is also orthogonal to the worldline. (b) In an arbitrary choice of synchrony, the time component e_0 is not orthogonal to the others, nor tangent to the worldline. Of the two possible choices of surface for constructing geodesics, we take that orthogonal to the worldline (and so not necessarily having no time component).

electromagnetic radiation, where the observer assigns coordinates to only those events which he sees. An advantage of the use of the null geodesics over the orthogonal geodesics would be that while orthogonal geodesics intersecting an accelerated observer's world-line at different events eventually intersect, even in flat space–time, and thus limit the validity of the coordinate system to a region around the world-line [143], null geodesics will not cause this limitation. However, the use of null geodesics in this approach results in an inconsistency: the connection coefficients are singular along the world line (see [224]). (However, for an inertial observer, coordinates may be assigned along null geodesics because then all the connection coefficients are zero along P. The time coordinate depends on the one-way speed of light along the geodesics and a locally flat metric is obtained.)

This leaves the choice of geodesics orthogonal to the world-line as the only one to give tractable results. Again we use the results of Sections 4.3 and 4.2.

Orthonormality of a geodesic with the world-line at P(t) requires that for the geodesic tangent vector V, $V \cdot e_0 = 0$. So

$$V^0 = -\kappa_n V^n \,, \tag{99}$$

(where now κ_n is $-f_{,n}(0)$ i.e. at the point P) and the coordinates along the geodesics are, from Eq. (89),

$$x^n = \lambda V^n, \qquad x^0 = ct + f(x^q) \,. \tag{100}$$

Using the chain rule, Eqs. (100), (86) allow the connection coefficients $\Gamma^{\mu}{}_{nm}$ to be expressed in terms of the other connection coefficients $\Gamma^{\mu}{}_{\nu 0}$ which themselves have already been determined along P by the application of Eqs. (93), (97) to this situation:

$$\Gamma^m{}_{pn} = -(\Gamma^m{}_{p0}f_{,n} + \Gamma^m{}_{n0}f_{,p} + \Gamma^m{}_{00}f_{,p}f_{,n}) , \tag{101}$$

$$\Gamma^0{}_{pn} = -(\Gamma^0{}_{p0}f_{,n} + \Gamma^0{}_{n0}f_{,p} + \Gamma^0{}_{00}f_{,p}f_{,n} + f_{,pn}) .$$

From Eqs. (93), (97), the connection coefficients along the world-line are then

$$(\Gamma^\mu{}_{\nu 0})^{\mathrm{t}} = \begin{pmatrix} -\chi_1\kappa_1 & \chi_1 & 0 & 0 \\ \chi_1(1-\kappa_1^2) - \chi_2\kappa_2 & \chi_1\kappa_1 & \chi_2 & 0 \\ -\chi_1\kappa_1\kappa_2 + \chi_2\kappa_1 - \chi_3\kappa_3 & \chi_1\kappa_2 - \chi_2 & 0 & \chi_3 \\ -\chi_1\kappa_1\kappa_3 + \chi_3\kappa_2 & \chi_1\kappa_3 & -\chi_3 & 0 \end{pmatrix} . \tag{102}$$

Although the coefficients $\Gamma^\mu{}_{\nu 0}$ are known along P by reason of tetrad propagation, they are not known along the orthogonal geodesics at this stage; their values might be obtained by integrating the expression for the Riemann tensor in terms of the connection coefficients and their derivatives [132]. From the definition of the connection coefficients, one could then obtain the basis vectors exactly along all the geodesics. Unfortunately the solution for the connection coefficients from the Riemann tensor expression is generally intractable (though not in the case of an inertial observer [132]).

The method used by Misner et al. [143] at this stage consists of solving the differential equations which result from expressing the connection coefficients in their Christoffel symbols form and does not readily lend itself to space-varying synchrony because the connection coefficients are evaluated at only one spatial point. Therefore an approximation must be used to obtain values for the basis vectors away from the world-line. Instead of making an approximation to the $\Gamma^\mu{}_{\nu 0}$ and thus the other connection coefficients, one can make an appropriate approximation to the covariant derivative which would require only the values of the connection coefficients along the world-line. Metric tensor components can then be found by taking inner products of the resultant basis vectors.

The basis vectors, $\{g_\mu\}$, along the orthogonal geodesics are obtained by parallel transport of the set $\{e_\mu\}$ along these geodesics (see [224]). They can be written in the following form which is valid only near the world-line:

$$g_0 = (1 + x^1\chi_1)e_0 - R^n(e_n - \kappa_n e_0)/c , \qquad g_n = f_{,n}g_0 + e_n - \kappa_n e_0 , \tag{103}$$

where the following definitions have been used.

$$R^1 = cx^2\chi_2 , \qquad R^2 = c(x^3\chi_3 - x^1\chi_2) , \qquad R^3 = -cx^2\chi_3 , \qquad R^n \equiv \varepsilon^{njk}x_j\Omega_k , \tag{104}$$

and where Ω is the observer's angular four-velocity, as mentioned in Section 4.3. (In this last equation, the observer's self-measured spatial angular velocity has components $\{\Omega^k\} = c\{\chi_3, 0, \chi_2\}$.)

The metric tensor components are obtained by taking inner products of the basis vectors Eq. (103) and using the relations in Eq. (79):

$$g_{00} = -(1 + x^1\chi_1)^2 + R^2/c^2 , \tag{105}$$

$$g_{0m} = -f_{,m}g_{00} - R^m/c , \tag{106}$$

$$g_{mn} = f_{,m}f_{,n}g_{00} + f_{,m}R^n/c + f_{,n}R^m/c + \delta_{mn} . \tag{107}$$

These metric components ignore curvature effects on the manifold because they do not take geodesic deviation into account. For a manifold with non-zero curvature, curvature effects come into play at second order [143], and so Eqs. (105)–(107) are, in general, valid only to first order. However, in a flat space-time theory such as special relativity they are in fact exact, agreeing with the metric derived by Nelson [150], Hehl et al. [83] and Scorgie [191], all of whom assume an orthonormal basis (when $f=0$). These results will be used in Section 5.2.2.

5. Tests of local Lorentz invariance

5.1. Review of alternative strategies of tests of local Lorentz invariance

Several different strategies have been proposed for testing for possible deviations from general and from special relativity. Much hinges on the choice or otherwise of a metric theory.

All metric theories of gravity are based on the Einstein equivalence principle, which assumes both the weak equivalence principle and local Lorentz invariance [17]. A metric theory possesses a symmetric, locally Lorentzian metric, with test particles following geodesics of that metric and with non-gravitational laws of physics being those of special relativity in local Lorentz frames. On a manifold this corresponds to the existence of a torsion-free affine connection, compatible with a Lorentzian metric of signature (1,3). The existence of local Lorentz invariance in a theory depends only on the nature of the affine connection and of the metric structure.

Other structure is needed to impart the full character of the corresponding space–time. Tests which honour special relativity (or local Lorentz invariance) within an alternative to general relativity (or Einstein's theory of gravity), are often performed in the context of the well-established parametrised post-Newtonian (PPN) framework of metric test theories [143, 231]. In this, just such other structural features, e.g. a revised role of the stress-energy tensor, different constants for the bending of space–time by matter and their possible dependence on a preferred frame, are used. In testing local Lorentz invariance, we are discussing a more radical departure in which preferred-frame effects may show up even within non-gravitational experiments and which therefore require a non-metric theory.

The most prominent tests of local Lorentz invariance are those in the tradition of, e.g., Gabriel and Haugan [64] Chupp et al. [31], Audretsch et al. [10], Haugan and Kauffmann [79]. Until recently (e.g. [167, 115] such experiments were cited as testing the isotropy of space, by which was meant testing for a preferred-frame directionality on atomic properties. This has now been more helpfully re-presented as testing local Lorentz invariance.

The strategy employed has currently reduced to a small choice of procedures. Either the Lagrangian is adapted, or a preferred-frame metric (or generic distance measurement) is proposed, on the basis of some non-metric test theory whose parameters enter the problem through this choice of dynamics or reference space. A fundamental equation of atomic physics such as the Dirac equation, written in a covariant formulation, is then solved using this Lagrangian and/or metric. The parameters of the test theory appear in its observables and therefore can be bounded by experiment.

By now, there are several choices of test theories to some extent chosen or adapted with an eye to the nature of the experimental test in hand. The $TH\varepsilon\mu$ theory postulates that the limiting speed of matter is not that of light, and adapts the material Lagrangian accordingly [64]. Haugan and Kauffmann [79] give a mature discussion as to how Moffat's nonsymmetric gravitation theory is a member of Ni's non-metric χ–g family of theories in which gravitational effects make space optically birefringent.

Tests of the principle of equivalence essentially require a non-metric theory as a test theory. For example, Nordtvedt [157] discusses the link between violations of gravitational red-shift and violations of the universality of free fall within non-metric theories. Opat and Unruh [161] suggest the comparison of atomic clocks at sea level at different latitude; this compares the effect of gravity on clock atoms and seawater to 1 part in 10^{18}. They suggest that a search for solar and lunar modulation of the frequencies may also be worthwhile. Such an experiment seems basically electromagnetic; pulses from each clock are sent to the other, and the relative lag is monitored.

Berglund et al. [17] discuss literature in which a preferred metric may affect the form of the weak interaction. In their case the possible coupling between magnetic field and particle momentum (expressed in a preferred-frame basis) is considered as it affects the precessional frequency of atoms in a magnetometer.

5.2. *Affine test theory with preferred vector field*

5.2.1. *Introduction*

The background for our own approach has been detailed in Section 3.4, and the context in Section 4.1. So far, we have concentrated on test theories in arbitrary synchrony for inertial frames in flat space–time. The Mansouri–Sexl test theory was formulated for inertial frames only. Generalizations by Abolghasem et al. [1,2] for the analysis of non-inertial observers applied a coordinate transformation for constant rotational (Earth) motion. Golestanian et al. [69] considered minimal deviations from the geometry of standard relativity. In addition, recent precision observation [7,203] of electromagnetic effects such as the Sagnac effect in ring lasers which are derived from (the non-inertial) Earth rotation again [200] raise the question as to what is learned about relativity from searches for a diurnal component of the Sagnac signal. These examples illustrate the value of extending our discussion of test theories and of the inclusion of arbitrary synchrony to an accelerated observer in a curved space–time.

The approach presented here (following [224]) has a geometrical foundation from which a family of coordinate transformations is derived, as opposed to simply being postulated. General non-inertial motions of an observer can be accommodated, giving a theory which contains the (synchrony-generalised) Mansouri–Sexl transformations [133] as a subclass. The geometric perspective proves to be complementary to the traditional approach in the Mansouri–Sexl formalism. Beil [14] quotes this as one illustration of non-metric-preserving transport, and shows how it is merely one class of a very much larger set of possible theories, much as the coordinate transformation investigated in this review is just one of a very much larger class of possibilities [81].

The use of general synchrony allows us in particular to revise one point of interpretation of Mansouri and Sexl by deriving the effect of synchrony on the parameter ε of the Mansouri–Sexl theory. Once the synchrony-independence of the physical predictions has been established, it is convenient to assume Einstein synchrony locally.

Several generalizations are possible. The coordinate transformations here can be used more generally, so as to transfer whatever physics is assumed in the preferred frame (say in neutron interferometry) to another frame. The assumption of flat space–time is unnecessary, since the affine structure is quite adequate to include curvature.

When special relativity is formulated on a four-dimensional manifold, its characteristic kinematics derives from the existence of two geometric structures on the manifold: an affine connection or law of vector transport corresponding in this case to a flat space–time, and a Lorentzian metric of signature (1,3) which is compatible with the connection and with invariance of the interval. In a standard construction, and with the use of orthonormal basis vectors for space–time coordinatization (tetrads) for all observers, inertial coordinates can then be found in which the connection coefficients are zero and the metric tensor components have the familiar orthogonal Minkowskian form $\{\eta_{\mu\nu}\} = \mathrm{diag}\{-1,1,1,1\}$. The invariance group of transformations which preserves this form of the metric components contains the Lorentz transformations, which then form the group of transformations from one set of inertial coordinates to another [62]. If the restriction to orthonormal bases is relaxed, for example by introducing a synchrony change, the metric components, the inner products of the basis vectors, no longer keep their diagonal form and the connection coefficients are not in general zero.

If an affine connection is torsion-free, there always exist coordinates centred on any event such that at that event the connection coefficients (but not necessarily their derivatives) are all zero [99] and then a (1,3) metric compatible with that connection is Lorentzian at that event. These properties hold, for example, for all PPN (parametrised post-Newtonian) test theories for general relativity.

To test local Lorentz invariance, one must go outside the restrictions of such theories, removing structure which corresponds to local Lorentz invariance, and operate in a framework of more general theories. Since kinematical analysis requires the comparison of vectors propagated along curves, an affine structure is needed, and so it is natural rather to choose to remove the metric as a geometric structure in order to produce a test theory.

The omission of a metric as a geometric structure on the manifold is not in conflict with relativistic kinematics, nor is it a denial of the existence of a space–time metric. Rather, it allows the test theory to examine both those theories which have a metric and those which do not, with the aim that experimental tests be used to restrict these theories and to determine the validity of local Lorentz invariance at a given level of precision or confidence.

On a general Riemannian manifold, the metric and the affine structures select distinct classes of geodesic. A metric structure singles out the curves of extremal distance, while the affine connection selects those curves which parallel transport their tangent vectors. These two classes coincide only if the connection is torsion-free and metric-compatible [84]. A theory with only an affine structure possesses geodesics which naturally correspond to unaccelerated motion and whose existence does not demand a metric structure. However, a formalism with only affine structure does not suffice for a test theory; it is too general to have the necessary predictive power and falsifiability.

In accord with, and to be as close as possible to, the original motivation of the Mansouri–Sexl test theory of Section 3.1, we propose a family of test theories in which the general affine structure is restricted by postulating an "aether"; that is, extra structure corresponding to a preferred frame is added. A natural candidate for the preferred frame is the cosmic background radiation, as discussed in Section 3.1.1. The philosophy of this approach is not so much a test for such a preferred frame as a test of local Lorentz invariance. After all, special relativistic kinematics is compatible with a preferred frame if the intrinsic preference does not affect the metric and frame transformation.

From a geometric perspective, the most natural way to impose this structure is to postulate the existence of a preferred vector field, X, whose integral curves are geodesic on the entire manifold.

X may be interpreted as the four-velocity of the preferred frame at each event, and the integral curves of X model the world-lines of the spatial points of the preferred frame.

The structure for the test theory is thus a torsion-free affine connection and the preferred vector field, X. To set up a coordinate system, an observer forms a set of basis vectors at each point of that system by simply taking partial derivatives with respect to the coordinates. Then that observer may define an inner product rule for these basis vectors, and thus define a metric. This has no physical meaning and does not suggest a metric for the manifold, but is rather a matter of description, which may or may not be invariant under a coordinate transformation. For mathematical convenience, it is assumed that each observer who propagates a tetrad defines a special relativistic type inner product relation similar to Eq. (79) between the basis vectors in that tetrad, and thus the formalism of Sections 4.2–4.4 can be used here. Note, however, that while such an inner product definition does define a sense of orthogonality, it makes no sense to talk about a metric in an observer's space.

While no space–time metric is discussed, this is independent of the existence of a metric in physical three-space; it is assumed that the physical three-space of each inertial observer in the theory is Euclidean. It is assumed that light travels along geodesics of the connection and furthermore, that in the preferred frame, Σ, the return-trip speed of light is isotropic (having value c) to first order.

5.2.2. *Development with arbitrary synchrony*

Consider an observer S who is in a laboratory frame in arbitrary motion, and let S's world-line be P, parameterised by his proper time t. Suppose that S defines a set of basis vectors, e_μ say, along P and let S define the inner product relation Eq. (79) along P. Such a definition in this context is purely mathematical and not necessarily related to any intrinsic property on the manifold. S can then assign the coordinates $\{x^\alpha\}=\{ct,x,y,z\}$ given in Eqs. (100), and his basis vectors g_μ near P are then given by Eqs. (103). Thus S formally has the same set of basis vectors and coordinates as the noninertial observer discussed in Section 4.4.

Writing

$$L=1+x^1\chi_1+R^n\kappa_n/c\,, \tag{108}$$

one can express S basis vectors in Eq. (103) as

$$g_0=L-R^n e_n/c\,,\qquad g_n=-(\kappa_n+f_{,n}L)e_0+(\delta_n{}^m+f_{,n}R^m/c)e_m\,. \tag{109}$$

Now, consider the preferred frame, which is represented by a vector field X. Any geodesic which is an integral curve of X can be used to represent the world-line of spatial point fixed with respect to the preferred frame. Let Σ be an observer at rest in the preferred (inertial) frame. Choose a geodesic integral curve, Π say, of X which intersects P at $P(0)$ and set the parameter, τ, of Π to be zero at $P(0)$. This corresponds to choosing the spatial origins of both frames to coincide when $t=\tau=0$.

Σ can define a coordinate system $\{\xi^\alpha\}=\{c\tau,\xi,\eta,\zeta\}$, in a manner similar to S, although because Σ is inertial the system will be simpler. Denote the basis vectors of Σ near Π by G_μ, with those along Π labelled $\mathscr{E}_\mu$, and let Σ define the following inner products and coordinates along a geodesic with tangent vector $\mathrm{d}/\mathrm{d}\lambda = V^\alpha \mathscr{E}_\alpha$, perpendicular to Π:

$$\langle \mathscr{E}_0|\mathscr{E}_0\rangle = -1\,, \qquad \langle \mathscr{E}_0|\mathscr{E}_m\rangle = -\kappa^0_m\,, \qquad \langle \mathscr{E}_m|\mathscr{E}_n\rangle = \delta_{mn} - \kappa^0_m\kappa^0_n\,,$$

$$G_0 = \mathscr{E}_0\,, \qquad G_n = h_{,n}\mathscr{E}_0 + \mathscr{E}_n - \kappa^0_n\mathscr{E}_0\,, \tag{110}$$

$$\xi^n = \lambda V^n\,, \qquad \xi^0 = ct + h(\{x^q\})\,, \tag{111}$$

where h and $\kappa^0_i \equiv -h_{,i}(0)$ are the counterparts in Σ of f and κ_i in S.

None of these steps requires a global metric; in particular, orthogonality along world-lines is arbitrarily defined.

Let E_μ be Σ's basis vectors along P, the world-line of S. At an event Q, lying a distance λ along a geodesic through $P(t)$, Σ's basis vectors G_μ are given in terms of E_μ by

$$\begin{aligned} G_0 &= E_0\,,\\ G_n &= E_n - [h_{,n}(Q) - h_{,n}(P(t))]\,E_0\\ &\equiv E_n - \Delta h_n E_0\,. \end{aligned} \tag{112}$$

The basis vectors E_μ and e_μ at $P(t)$ are related by some transformation:

$$E_\mu = T_\mu{}^\nu e_\nu\,. \tag{113}$$

This leads to (details are given in [224])

$$L\,\mathrm{d}t - (\kappa_m + f_{,m}L)\,\mathrm{d}x^m = (c\,\mathrm{d}\tau - \mathrm{d}\xi^n\Delta h_n)(1/Y^0 - v^n T_n{}^0) + \mathrm{d}\xi^n T_n{}^0 \tag{114}$$

and

$$-R^m\,\mathrm{d}t + \mathrm{d}x^n(\delta_n{}^m + f_{,n}R^m/c) = b^m{}_p(\mathrm{d}\xi^p - (c\,\mathrm{d}\tau - \mathrm{d}\xi^n\Delta h_n)v^p/c)\,, \tag{115}$$

$$b^m{}_p \equiv T_p{}^m \tag{116}$$

where $\{b^m{}_p\}$ is assumed to be an invertible matrix.

When S is inertial ($\chi_1 = \chi_2 = \chi_3 = 0$), and when $h = -\kappa^0\cdot\xi$ and $f = -\boldsymbol{\kappa}\cdot\boldsymbol{x}$, one has $R^n = 0$, $\Delta h_n = 0$, and $f_{,n} = -\kappa_n$. Identifying $a = 1/Y^0$, $\varepsilon_m = T^0_p(b^{-1})^p{}_m$, Eqs. (114), (115) give the synchrony-generalised Mansouri–Sexl test-theoretic result of Eqs. (54), (55). a is the time dilation parameter, $\boldsymbol{\varepsilon}$ depends on the synchrony choice in both frames, and $b^m{}_p$ are length contraction parameters.

It is interesting to note the significance of $\boldsymbol{\varepsilon}$. From Eq. (113), if $T^0_p = 0$ then, at any point along P, the S spatial basis vectors span the same surface as do Σ's spatial basis vectors. In the context of the Mansouri–Sexl test theory [133] (flat space–time) and because $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}^0$ are constant, Σ and S share the same foliation of space–time along P. Thus they agree on whether two events are simultaneous or not. This perspective explicitly shows the conventional nature of simultaneity, and is in accord with Mansouri–Sexl test theory where $\boldsymbol{\varepsilon}$ was introduced as a measure of the difference in time intervals between Σ and S. For zero $\boldsymbol{\varepsilon}$ there is agreement on simultaneity, but not necessarily on time intervals.

An expression for $\boldsymbol{\varepsilon}$ may be obtained in terms of the other parameters (details are given in [224]):

$$\varepsilon_m = -\kappa_m - a(b^{-1})^n{}_m \frac{-(1 + \boldsymbol{\kappa}^0\cdot\boldsymbol{v}/c)\kappa^0_n + v^n/c}{(1 + \boldsymbol{\kappa}^0\cdot\boldsymbol{v}/c)^2 - v^2/c^2}\,, \tag{117}$$

in agreement with the expressions for ε obtained using the operational approach of Section 3.2. When Einstein synchronization is imposed ($\boldsymbol{\kappa}=\mathbf{0}$), the above equation coincides with Eq. (71). This demonstrates that ε, while depending on a and b, is not a discriminator of theories, but reflects relative synchrony conventions. Hence, synchrony choice does not affect experimental predictions for measurables either within the Mansouri–Sexl test theory (see Section 3.2; the parameters a and b are measurable only within synchrony equivalence classes) or within our generalised theory.

Hence, for the purpose of making predictions, it is convenient to put $\boldsymbol{\kappa}=\mathbf{0}$ and $f=0$ to simplify analysis. This gives ε the simple form in Eq. (71). Such a choice is not in conflict with the conventionality of distant simultaneity as long as there is then no attempt to claim experimental distinction of this synchrony choice.

5.2.3. Test theory in Einstein synchrony

As just explained, we now set $\boldsymbol{\kappa}=\mathbf{0}$. The velocity of a point at $\boldsymbol{x}$ in S with respect to the instantaneously comoving, non-rotating frame, S_s, is $\boldsymbol{\Omega}\times\boldsymbol{x}=-\boldsymbol{V}$. So the spatial coordinates assigned to an event will be $\boldsymbol{x}=\boldsymbol{x}_s+\int_0^t \boldsymbol{V}\,\mathrm{d}t$, $\mathrm{d}\boldsymbol{x}-\boldsymbol{V}\,\mathrm{d}t=\mathrm{d}\boldsymbol{x}_s$.

In this manner we find the following form for the frame transformation from Eqs. (114), (115):

$$\mathrm{d}x^0 = a\frac{\mathrm{d}\xi^0-\mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v}/c^2}{(1+x^1\chi_1)(1-v^2/c^2)}, \tag{118}$$

$$\mathrm{d}\boldsymbol{x}-\boldsymbol{V}\,\mathrm{d}t=b(\mathrm{d}\boldsymbol{\xi}-\boldsymbol{v}\,\mathrm{d}\tau). \tag{119}$$

Hence although S is rotating with respect to Σ, b is not affected by the changing orientation of the two axis sets. This is because $\mathrm{d}\boldsymbol{x}-\boldsymbol{V}\,\mathrm{d}t$ has constant orientation with respect to a nonrotating frame and thus Σ. Hence, requiring that the space axes of S and Σ are parallel when $t=0$ ensures that the "length contraction" matrix b in Eq. (115) has no rotational components.

We now choose to simplify the form of the length contraction matrix b, by regarding it as a tensor under 3-space rotation, so making it feasible to choose the coordinate axes as its principal axes. We therefore separate the action of b into two parts by assuming that b acts on the direction $\boldsymbol{v}$ independently of its action in the plane perpendicular to $\boldsymbol{v}$. Furthermore, we assume that both directions perpendicular to $\boldsymbol{v}$ are acted on in the same way and that the action of b is purely to scale by factors β and δ in directions parallel and perpendicular to $\boldsymbol{v}$:

$$b\boldsymbol{w}=\begin{cases}\delta\boldsymbol{w} & \text{if } \boldsymbol{w}\perp\boldsymbol{v},\\ \beta\boldsymbol{w} & \text{if } \boldsymbol{w}\|\boldsymbol{v}.\end{cases} \tag{120}$$

From this equation and the identity $(\boldsymbol{v}\times\boldsymbol{p})\times\boldsymbol{v}=v^2\boldsymbol{p}-(\boldsymbol{v}\cdot\boldsymbol{p})\boldsymbol{v}$, we obtain the action of b on an arbitrary (spatial) vector:

$$b\boldsymbol{p}=\beta\frac{\boldsymbol{v}\cdot\boldsymbol{p}}{v^2}\,\boldsymbol{v}+\delta\frac{\boldsymbol{v}\times\boldsymbol{p}}{v^2}\times\boldsymbol{v}. \tag{121}$$

Since the effect of b can be broken down into the sum of independent scaling effects, it follows that the inverse action of b is given by

$$b^{-1}\boldsymbol{p}=\frac{1}{\beta}\frac{\boldsymbol{v}\cdot\boldsymbol{p}}{v^2}\,\boldsymbol{v}+\frac{1}{\delta}\frac{\boldsymbol{v}\times\boldsymbol{p}}{v^2}\times\boldsymbol{v}. \tag{122}$$

Restating Eq. (118), and using Eq. (121) in Eq. (119), gives

$$\mathrm{d}t = a\frac{\mathrm{d}\tau - \mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v}/c^2}{(1+x\chi)(1-v^2/c^2)}\,, \tag{123}$$

$$\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t = \left(\beta\frac{\mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v}}{v^2} - \mathrm{d}\tau\right)\boldsymbol{v} + \delta\left(\frac{\boldsymbol{v}\times\mathrm{d}\boldsymbol{\xi}}{v^2}\right)\times\boldsymbol{v}\,, \tag{124}$$

where β, δ and a are functions of $\boldsymbol{v}$, and χ_1 has been written as χ. These last two equations are the final form of the transformations of the test theory for the case where all observers propagate orthonormal tetrads.

When the special relativistic values of the three parameters ($a = 1/\gamma_v$, $\beta = \gamma_v$ and $\delta = 1$) are substituted into Eqs. (123) and (124) the resulting transformation is in agreement with that of Nelson [150] for the coordinate transformation from an arbitrarily accelerating frame to an inertial frame within special relativity. This can be seen by taking infinitesimals of Eq. (19) of [150].

An expression for the one-way speed of light for S, corresponding to the choice $f = 0$, is obtainable by using the value chosen in Σ's frame and transforming to the S-frame. Eqs. (119) and (122) give

$$\mathrm{d}\boldsymbol{\xi} - \boldsymbol{v}\,\mathrm{d}\tau = \frac{\boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\mathrm{d}t)}{\beta v^2}\boldsymbol{v} + \frac{\boldsymbol{v}\times(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)}{\delta v^2}\times\boldsymbol{v}\,, \tag{125}$$

$$\mathrm{d}\boldsymbol{\xi}\cdot\boldsymbol{v} - v^2\,\mathrm{d}\tau = \boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)/\beta\,. \tag{126}$$

From here, using Eq. (123),

$$\mathrm{d}\tau = \frac{1+x\chi}{a}\mathrm{d}t - \frac{\boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)}{\beta(c^2 - v^2)}\,. \tag{127}$$

Using this and Eq. (125),

$$\mathrm{d}\boldsymbol{\xi} = \left(\frac{\boldsymbol{v}\cdot(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)}{\beta v^2(1-v^2/c^2)} + \frac{1+x\chi}{a}\mathrm{d}t\right)\boldsymbol{v} + \frac{\boldsymbol{v}\times(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)}{\delta v^2}\times\boldsymbol{v}\,. \tag{128}$$

These two results for $\mathrm{d}\tau$ and $\mathrm{d}\xi$ then give

$$\mathrm{d}\xi^2 - c^2\,\mathrm{d}\tau^2 = \frac{(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)^2}{\delta^2} - c^2\frac{(1+x\chi)^2\,\mathrm{d}t^2}{a^2\gamma_v^2} + \frac{[(\mathrm{d}\boldsymbol{x} - \boldsymbol{V}\,\mathrm{d}t)\cdot\boldsymbol{v}]^2}{v^2}\left(\frac{\gamma_v^2}{\beta^2} - \frac{1}{\delta^2}\right). \tag{129}$$

Since the one-way speed of light in Σ has been chosen as c, in S its value in a direction $\hat{\boldsymbol{p}}$ is given by putting the left hand side equal to zero in Eq. (129) and solving for $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = c_p\hat{\boldsymbol{p}}$:

$$\frac{c_p^2}{c^2}\left(\frac{1}{\delta^2} + \frac{(\boldsymbol{p}\cdot\boldsymbol{v})^2}{v^2}\left(\frac{\gamma_v^2}{\beta^2} - \frac{1}{\delta^2}\right)\right) - 2\frac{c_p}{c}\left(\frac{\boldsymbol{p}\cdot\boldsymbol{V}}{c\delta^2} + \frac{(\boldsymbol{p}\cdot\boldsymbol{v})(\boldsymbol{V}\cdot\boldsymbol{v})}{cv^2}\left(\frac{\gamma_v^2}{\beta^2} - \frac{1}{\delta^2}\right)\right) \tag{130}$$

$$= \frac{(1+x\chi)^2}{a^2\gamma_v^2} - \frac{R^2}{c^2\delta^2} - \frac{(\boldsymbol{V}\cdot\boldsymbol{v})^2}{c^2v^2}\left(\frac{\gamma_v^2}{\beta^2} - \frac{1}{\delta^2}\right). \tag{131}$$

Note that when β and δ take on their special relativistic values all velocity dependence in the above equation vanishes.

While the present formalism seems to be kinematic, dynamical considerations (such as electromagnetic or gravitational effects) are latent, and imply the postulation of dynamical behaviour. The most natural way to insert this is to require a particular dynamical behaviour in the preferred frame and to transform this to the test frame via Eqs. (123), (124).

5.3. Sagnac effect

A novel feature of experimental laser research is the steady improvement over decades in the accuracy with which noninertial effects are measured. The Sagnac effect has now been seen in a wide variety of interferometers [7] including, in particular, SQUIDs or superconducting Cooper pair interferometers [239] optical interferometers, neutron interferometers [36] and most recently electron and atomic interferometers [77]. Large ring laser experiments are presently earth-bound and so inevitably are rotating with respect to the local Lorentz frame. The detection of the Sagnac effect arising from the rate Ω_E of rotation of the earth was initially performed optically [141], and the detection by small ring lasers predated the vivid and better-documented demonstration by neutron interferometry [201]. By now, several ring laser systems are reported to have detected the associated Sagnac effect (for example, see [202, 124, 125]). In the Canterbury ring laser, the frequency resolution can reach 10^{-6} of that from the earth-rotation-induced Sagnac effect [202], and it seems to be feasible to detect the secular variations in the rate of the rotation of the earth (at the level of $10^{-8}\Omega_E$) in a somewhat larger device. Optical interferometry still leads the field for relative accuracy in such a measurement. In addition, several studies of ring lasers under significant acceleration have been reported [112, 30, 59] and some elegant experiments by Kowalski et al. [101, 102] explore at novel precision the effect of acceleration or of gravity, applied to some or all of the optical components of the ring laser system, on the beat frequency of ring lasers containing dielectrics.

The nature of ring interferometric effects within a preferred frame theory and the potential of ring lasers in bounding deviation from local Lorentz invariance can be seen from the analysis given in Section 5.4 for a vacuum ring.

5.4. Ring laser tests

In the context of the test theory of Section 5.2.1, with the imposition $h=0$ for Σ (corresponding to the choice of an isotropic one-way speed of light for Σ), the parameters a, β, and δ are expandable in terms of v^2 (see [133]). Thus it is seen from Eq. (131) that the expression for the one-way speed of light in S has only even powers of $\boldsymbol{v}$ in a velocity expansion. Hence any closed-loop optical test covered by the test theory in Section 5.2.1, and hence covered by the theory of Mansouri and Sexl, can at best be of second order in the preferred frame velocity. While some choices of synchrony in S would introduce first-order velocity dependence in the one-way speed of light in S, this dependence would cancel out over a closed path (because of the covariance of the formalism). Similarly, a choice other than Einstein synchronization in Σ (while resulting in odd powers of v in the expressions for the test theory parameters) will have the same effect in S.

Within this test theory, an analysis of the Sagnac effect in a ring laser predicts sidereal, $\boldsymbol{v}$-dependent variations in the measured beat frequency. Following Scorgie [190], consider an arbitrary, smooth closed path $\mathscr{C}$ along which laser beams travel in both senses. Denote T_+ and T_- as the times taken for light to traverse $\mathscr{C}$ in anti-clockwise and clockwise senses respectively and let c_+ and c_- be the (position-dependent) speed of light in those respective senses. Taking $\mathrm{d}l$ to be an element of arc along $\mathscr{C}$ in an anti-clockwise sense (so that $c_+\boldsymbol{p}=\mathrm{d}\boldsymbol{l}/\mathrm{d}t$), one has, from Eq. (131), the following expression for the difference in transit time for the two directions:

$$T_+ - T_- = \oint_{\mathscr{C}} \left(\frac{1}{c_+} - \frac{1}{c_-}\right) \mathrm{d}l \tag{132}$$

$$= 2 \oint_{\mathscr{C}} \frac{(\boldsymbol{V} + (\boldsymbol{V}\cdot\boldsymbol{v})Q\boldsymbol{v}/v^2)\cdot \mathrm{d}\boldsymbol{l}}{R^2 + Q(\boldsymbol{V}\cdot\boldsymbol{v})^2/v^2 - (c\delta(1+x\chi)/a\gamma)^2}\,, \tag{133}$$

where $Q \equiv \delta^2\gamma^2/\beta^2 - 1$.

For an Earth-bound ring laser having a rotation of the order of the Earth's, the linear acceleration is due to gravity, is small, and contributes negligibly in Eq. (132). Similarly the Earth's small angular rotation magnitude Ω allows $\boldsymbol{V}=\boldsymbol{x}\times\boldsymbol{\Omega}$ to be ignored at second order.

Using Eq. (104), Stokes' theorem and the vector identities

$$\nabla\times(\boldsymbol{x}\times\boldsymbol{\Omega}) = -2\boldsymbol{\Omega}\,, \qquad \nabla\times[\boldsymbol{x}\times(\boldsymbol{\Omega}\cdot\boldsymbol{v})\,\boldsymbol{v}] = (\boldsymbol{\Omega}\cdot\boldsymbol{v})\boldsymbol{v} - v^2\boldsymbol{\Omega}\,, \tag{134}$$

and assuming that the change in $\boldsymbol{v}$ is negligible over the time it takes a light signal to traverse $\mathscr{C}$ (so that some of the v-dependent parameters may be treated as constants under integration), Eq. (132) becomes:

$$T_+ - T_- = -\frac{2a^2\gamma^2}{\delta^2c^2} \oint_{\mathscr{C}} \left(\boldsymbol{V} + Q\frac{\boldsymbol{V}\cdot\boldsymbol{v}}{v^2}\boldsymbol{v}\right)\cdot \mathrm{d}\boldsymbol{l}$$

$$= \frac{2a^2\gamma^2}{\delta^2c^2}\left[(2+Q)\boldsymbol{\Omega}\cdot\boldsymbol{A} - Q\frac{(\boldsymbol{\Omega}\cdot\boldsymbol{v})(\boldsymbol{v}\cdot\boldsymbol{A})}{v^2}\right]. \tag{135}$$

When a, β and δ take on the special relativistic values ($1/\gamma$, γ and 1, respectively), with $Q=0$ as a result, Eq. (135) becomes

$$T_+ - T_- = \frac{4\boldsymbol{\Omega}\cdot\boldsymbol{A}}{c^2} \tag{136}$$

which is the standard result for special relativity. Such a transit time difference in an interferometer translates into a frequency difference Δf_b between counter-propagating waves in an active ring laser [201] $\Delta f_b = c^2(T_+ - T_-)/\lambda_0 P_0$ where λ_0 is the laser wavelength (the HeNe value 633 nm in the case considered below) and P_0 the perimeter of the ring (3.477 m). This then gives in special relativity $\Delta f_b = 4\boldsymbol{\Omega}\cdot\boldsymbol{A}/\lambda_0 P_0$.

Because we are now working within the Einstein synchrony choice, a, β and δ are even functions of v [133, I]. From here we work to second order in v/c, when $a = 1 + a_2v^2/c^2$, $\beta = 1 + \beta_2v^2/c^2$, $\delta = 1 + \delta_2v^2/c^2$ (in special relativity $a_2 = -\frac{1}{2}$, $\beta_2 = \frac{1}{2}$, $\delta_2 = 0$). Hence

$$\frac{a^2\gamma^2}{\delta^2} = 1 + (1 + 2(a_2 - \delta_2))\frac{v^2}{c^2}, \qquad Q = (1 + 2(\delta_2 - \beta_2))\frac{v^2}{c^2}\,, \tag{137}$$

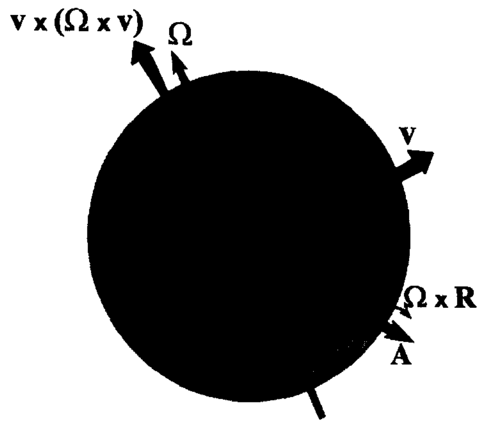


Fig. 11. If the preferred frame velocity $\boldsymbol{v}$ is taken as that relative to the cosmic microwave background, the test-theory described here permits the projection of $\boldsymbol{v}\times(\boldsymbol{\Omega}\times\boldsymbol{v})$ on the ring area to be observable as a diurnal variation in beat frequency.

and so (using Eq. (134)), with $\boldsymbol{X}\equiv(\boldsymbol{v}\times\boldsymbol{\Omega})\times\boldsymbol{v}$:

$$T_+ - T_- = 4\left[1+(1+2(a_2-\delta_2))\frac{v^2}{c^2}\right]\frac{\boldsymbol{\Omega}\cdot\boldsymbol{A}}{c^2} + 2\,[1+2(\delta_2-\beta_2)]\,\frac{\boldsymbol{X}\cdot\boldsymbol{A}}{c^4} \tag{138}$$

$$= 4\left[1+\left[\tfrac{3}{2}+2a_2-\delta_2-\beta_2\right]\frac{v^2}{c^2}\right]\frac{\boldsymbol{\Omega}\cdot\boldsymbol{A}}{c^2} - 4\left[\tfrac{1}{2}+\delta_2-\beta_2\right]\frac{(\boldsymbol{\Omega}\cdot\boldsymbol{v})(\boldsymbol{v}\cdot\boldsymbol{A})}{c^4} \tag{139}$$

We take $\boldsymbol{v}=\boldsymbol{v}_0+\delta\boldsymbol{v}$ where $\boldsymbol{v}_0$ is the velocity of the (centre of the) Earth with respect to the preferred frame defined by the cosmic microwave background (CMB) [57] and $\delta\boldsymbol{v}=\boldsymbol{\Omega}\times\boldsymbol{R}$ and is that due to Earth rotation (like $\boldsymbol{R}$, relative to the centre of the Earth) of the ring laser laboratory at $\boldsymbol{R}$, Cashmere, Christchurch, New Zealand (latitude $\lambda = 43°34'37''$S) (Fig. 11).

In previous publications we have used the data quoted by Narlikar et al. [149], that the Earth's velocity with respect to the preferred frame is $v_0 = 2.4\times10^{-3}c$ at a declination δ of $-26°$. In view of the following more refined estimate both figures seem to be substantially incorrect. We now believe that Narlikar et al. used figures for the local group of galaxies, or the Milky Way, rather than those appropriate for Earth-bound experiments such as the ones they and we discuss. This, and especially the speed correction, will have some effect on their effort to reinterpret some laser experiments. Because $\boldsymbol{X}$ is only a few degrees removed from $\boldsymbol{\Omega}$, which has a constant projection on the ring area, it is important to make the relative estimates carefully.

We take here recent values of the galactic latitude l^{II} and longitude b^{II}, etc., also the magnitude of the dipole term and the resulting velocity (367 km/s, or $1.22\times10^{-3}c$) for the apex of the CMB dipole from Kogut et al. [100] and Bennett et al. [16] as in Table 1. Converting [240] the average values to equatorial coordinates gives a declination and right ascension of $\delta = -6.82\pm0.12$, $\alpha = 167.52\pm0.09 = 11^h.17$. This is on the boundary of the constellations Leo and Crater (the microwave photons seem hotter when from this direction), and approximately 10° S of the Ecliptic (Fig. 11). The declination δ of the CMB as seen from Earth is the angle between $\boldsymbol{v}_0$ and the Equatorial plane.

The term present in Eq. (138) when $v=0$ is the Sagnac effect predicted by (special and general) relativity [201]. On time averaging over one sidereal day the second term (that proportional to $a_2-\delta_2$) and also the third term introduce a bias on the nominal Sagnac value, changing the apparent magnitude and direction of the Earth rotation rate $\boldsymbol{\Omega}$. These biases will be undetectable in practice, being considerably smaller than the calibration errors of the area measurement for example. In addition, these terms exhibit sinusoidal variations with a period of a sidereal day because the rotation of the Earth changes the direction of $\delta\boldsymbol{v}$, inducing a related variation in the magnitude v^2 of the form $2\delta\boldsymbol{v}\cdot\boldsymbol{v}_0$. In addition, in sensing the projection of the vector $\boldsymbol{X}$ on the ring area $\boldsymbol{A}$, the third term is subject to the sinusoidal change in the angle of this projection and so will give another signal with a period of the sidereal day. It is required here for both mechanisms (v variation and $\boldsymbol{X}$ projection) that $\boldsymbol{v}$ is not parallel to $\boldsymbol{\Omega}$, and for the second ($\boldsymbol{X}$ projection) that $\boldsymbol{X}$ is not parallel to $\boldsymbol{\Omega}$. Because $\boldsymbol{X}\cdot\boldsymbol{A} = v^2\boldsymbol{\Omega}\cdot\boldsymbol{A}-(\boldsymbol{v}\cdot\boldsymbol{A})\,(\boldsymbol{v}\cdot\boldsymbol{\Omega})$, the first term in this expression for $\boldsymbol{X}\cdot\boldsymbol{A}$ can be incorporated with the earlier terms proportional to v^2, as in the first term in Eq. (139), giving a correction

$$C_1=\left[\frac{\Delta f'}{\Delta f_b}\right]_1=[3+4a_2-2\delta_2-2\beta_2]\,\frac{\boldsymbol{\Omega}\times\boldsymbol{R}\cdot\boldsymbol{v}_0}{c^2}\,.$$

The numerator of the second term in Eq. (139) contains a leading term $Y_0=(\boldsymbol{\Omega}\cdot\boldsymbol{v}_0)(\boldsymbol{v}_0\cdot\boldsymbol{A})$ which has a diurnal variation in principle; the direction of $\boldsymbol{A}$ varies as the Earth rotates. This gives a contribution (carets denoting unit vectors):

$$C_2=\left[\frac{\Delta f'}{\Delta f_b}\right]_2=-\left[\tfrac{1}{2}+\delta_2-\beta_2\right]\frac{(\boldsymbol{v}_0\cdot\hat{\boldsymbol{\Omega}})(\boldsymbol{v}_0\cdot\hat{\boldsymbol{A}})}{c^2\sin\lambda}\,.$$

This term will be greater in magnitude, because it does not depend on the rotation-induced speed $\delta v\le 465$ m/s of the Earth's surface at the ring laser laboratory. Physically then, this term reflects an anisotropy $\delta_2-\beta_2$ of length contraction (parallel and perpendicular to the relative velocity) additional to that $(\delta_0-\beta_0)$ defined by the Lorentz transformation, but formally permitted within the generalised Mansouri–Sexl test theory.

It is interesting to note in this connection that all such corrections from any such test theory in any experiment, including all those discussed in Section 3, are bilinear in speeds which have to be the relative speeds of physical material. Y_0 makes both of these velocity factors a speed typical of the solar system, $v_0=367$ km/s (Table 1). Any experiment to rival for precision the experiment we discuss has either to depend totally (i.e. doubly) on v_0 or has to rival this cosmic speed. This is not difficult since this speed is still much less than the speed of light: $v_0/c=1.22\times10^{-3}$; even a sodium

Table 1
Values for CMB dipole anisotropy from Kogut et al. [100] and Bennett et al. [16]. l^{II} and b^{II} are the galactic longitude and latitude; δT is the maximum temperature excess of the CMB, and v_0 the corresponding speed

	Kogut	Bennett
l^{II}	$264.4\pm0.3°$	$264.4\pm0.2°$
b^{II}	$48.4\pm0.5°$	$48.1\pm0.4°$
δT	3.365 ± 0.027 mK	$3.363+0.024$ mK
v_0	367 km/s	367 km/s

ion, for example, reaches this speed with 28 kV. One may argue with Narlikar et al. [149] for a cosmic identification of any preferred frame relevant for any laser experiment, when the relative velocities will be the same. What is unique is the requirement for a non-inertial frame in order to generate the Sagnac effect, including its possible preferred-frame dependence; and to describe the latter, the generalised test-theory given here is essential.

The next terms within the numerator of Eq. (139), $Y_1 = (\boldsymbol{\Omega}\cdot\delta\boldsymbol{v})(\boldsymbol{v}_0\cdot\boldsymbol{A}) + (\boldsymbol{\Omega}\cdot\boldsymbol{v}_0)(\delta\boldsymbol{v}\cdot\boldsymbol{A})$, might be thought to be comparable with C_1 (if not C_2) in that both these corrections (Y_1, C_1) are linear in each of the speed v_0 and the rotation-induced speed δv of the Earth's surface. However Y_1 vanishes because $\delta\boldsymbol{v} = \boldsymbol{\Omega}\times\boldsymbol{R}$ and so is perpendicular to both $\boldsymbol{\Omega}$ and $\boldsymbol{A}$ for a horizontal ring laser (when $\boldsymbol{A}$ is parallel to $\boldsymbol{R}$).

We estimate C_1 and C_2. With a suitable choice of time origin $\boldsymbol{v}_0\cdot\hat{\boldsymbol{A}} = v_0\cos(\delta+\lambda)\cos\Omega t$, $\boldsymbol{v}_0\cdot\hat{\boldsymbol{\Omega}} = v_0\sin\delta$, and $\boldsymbol{\Omega}\times\boldsymbol{R}\cdot\boldsymbol{v}_0 = v_0\Omega R\cos\lambda\cos\delta\sin\Omega t$. The two terms are therefore distinguishable experimentally through having phases differing by 90°. Although C_1 is four orders of magnitude smaller than C_2, the corresponding parameters have not been measured nearly as accurately, and this term should strictly be retained. For simplicity of exposition we keep only the dominant term, which will be of amplitude

$$\begin{aligned} C_2^{\max} &= [\tfrac{1}{2}+\delta_2-\beta_2]\left(\frac{v_0^2}{c^2}\right)\frac{\cos(\delta+\lambda)\sin\delta}{\sin\lambda} \\ &= 9.2\times10^{-4}[\tfrac{1}{2}+\delta_2-\beta_2]\,. \end{aligned}$$

If therefore we wish to detect deviations in $\delta_2-\beta_2$ from the special relativistic value (of $-\frac{1}{2}$) at say the level of parts per thousand, we need to monitor the Sagnac frequency signal ($\Delta f_b = 69$ Hz) for a diurnal fluctuation of relative magnitude 1 ppm, or an absolute magnitude of the order of 70 μHz. Current frequency resolution has already reached this goal [203], although a diurnal variation such as that proposed here would have been eliminated in current dedrifting techniques. However as Müller et al. [146] show in their recent and useful summary, other experiments have greatly improved on this, determining $\delta_2-\beta_2$ to parts in 10^9. Robinson [184] makes this even more precise in the light of studies of atomic or nuclear frequencies. The test considered here, then, is properly regarded as a test of the whole theoretical framework which leads to Eq. (131), with its considerable novelty over the expression standard for Mansouri–Sexl theory in inertial frames [146].

Acknowledgements

We are grateful to Professors B.J. Hunt, L. Kannenberg, D. Malament, G.I. Opat, D.K. Ross, Drs H.R. Brown, B. Ellis, D. Gunn, T.P. Krisher, J. Stachel, R. Zalaletdinov, also H.J. Ross and M.D. Hannam for discussions and/or correspondence.

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